Local Network Effects, Rationality and The Structure of Technology Networks

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How I got started on this research

How does one optimally price IT-based products?

- Unusual cost structure
- Threat of digital piracy
- Presence of (mostly) positive network effects

How I got started on this research

How does one optimally price IT-based products?

- Unusual cost structure
- Threat of digital piracy
- Presence of (mostly) positive network effects
  - Network effects depend on individual usage, may be heterogeneous in value across customers
  - *Nonlinear pricing and type-dependent network effects (2004)*
  - *Network effects, nonlinear pricing and entry deterrence (2005)*

Other aspects of network effects (that seem to matter)

- Agents often are not capable of (or interested in) forming rational expectations which are fulfilled
  - They don't have enough information about the preferences of other agents
  - They don't have the ability to compute a rational expectations equilibrium even if they did have the information
  - They don't pay attention to every product all the time
  - They base their beliefs on "local" information
- Adoption is often gradual and "viscous", rather than being instantaneous
  - The realized dynamic process of adoption often determines eventual outcomes

Other aspects of network effects (that seem to matter)

- Network effects are often "local"
  - Interpersonal communication technologies, business to business technologies, online marketplaces…
- The structure of underlying social or business networks affects the adoption of network goods
  - An agent's "local" network affects their value from adoption…
  - …but so does the structure of the rest of the social network
    - Local networks are connected
    - One's neighbors' local networks affect one's adoption
  - Structure of the "adoption network" (or technology network) depends on the structure of the underlying social network

My research questions

- How is the adoption of a technology which displays network effects affected by:
  - The extent to which the network effects are local
  - The extent to which their value differs across potential adopters
  - The structure of an underlying social or business network
  - The "boundedness" of consumer rationality
- What can one infer about each of these from the observed structure of an adoption network?
- What are the implications of a model of this kind for:
  - Optimal price paths for new network goods
  - Choosing how connected targeted early adopters should be
  - The benefits of mandated IT standards in an organization, or in an inter-organizational supply network
  - The social optimality of universal access to a technology

Models of networks: a framework(?)

Network effects in economics

- Network effects are homogeneous
- Network effects are "global"
- Social network is complete
- Agents are unboundedly rational
- Adoption is instantaneous
- Adoption cost is strategic (price)

"Science of networks" models

- Network effects are homogeneous
- Network effects are local
- Social network is complete
- Agents are myopic
- Adoption is gradual (discrete)
- Adoption cost is constant (zero)

Local network effects…

- **Network effects are heterogeneous**
- **Network effects are local**
- **Social network is any graph**
- **Agents are unboundedly rational**
- **Adoption is instantaneous**
- **Adoption cost is constant (but can be a one-shot strategic variable)**

Dynamic pricing of network…

- **Network effects are heterogeneous**
- **Network effects are "global"**
- **Social network is complete**
- **Agents are boundedly rational (myopic, stubborn, combination)**
- **Adoption is gradual (continuous)**
- **Adoption cost is strategic and varies over time**

Underlying social network: example

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"Adoption network": an example

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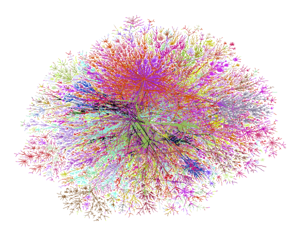
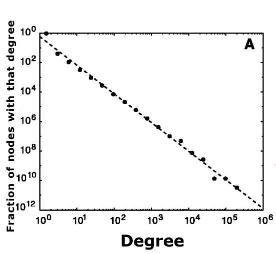
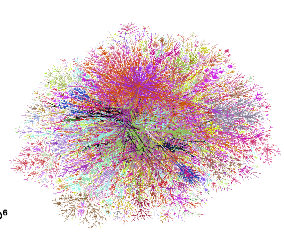
Adoption networks: another example

- **Degree** of a node: number of other nodes a node is connected to (or number of edges originating from the node)
- **Degree distribution** of a network: Fraction of nodes in a network that have a particular degree, as a function of degree



Adoption networks: another example





Degree distribution of the Web

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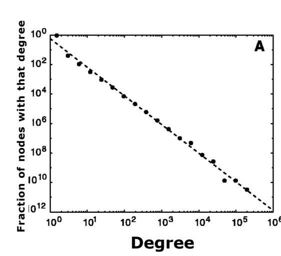
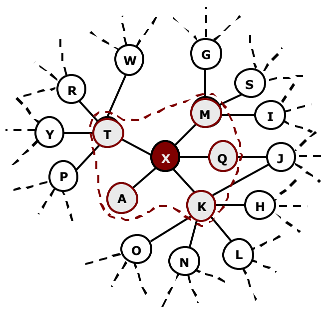
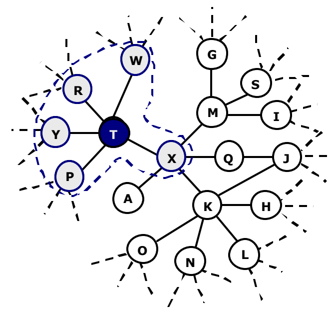


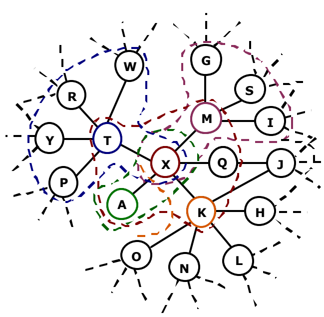
Local networks



Local networks



Local networks



Local network effects

- Agents in this kind of network generally have:
  - different local networks
  - perfect information about the structure of their local network
  - some information about the structure of the other local networks they belong to (their neighbors' local networks)
  - very little or no information about the exact structure of the rest of the social network
- These agents make their adoption decisions based on their local networks, and this information.

A model of local network effects

- Set of potential customers $N = \{1,2,3,\ldots,n\}$
- Single homogeneous network good that costs *c*
- Customers connected by an underlying social network modeled as an instance of a random graph (more on this soon).
- Each customer has:
  - A neighbor set G_i
  - A degree d_i (number of neighbors)
  - A valuation type θ_i (measure of adoption complementarity)
- Each customer makes an adoption choice $a_i \in \{0,1\}$
- Payoff from adoption for customer *i:*

$$a_i[u(\sum_{j \in G_i} a_j, \theta_i) - c]$$

- More generally formulated in the paper

Where the social network comes from

$N = \{1,2,3,\ldots,n\}$ $\quad \Gamma_i = 2^{N \setminus \{i\}}$

Set of graphs: $\Gamma \subset \Gamma_1 \times \Gamma_2 \times \ldots \times \Gamma_n$

Distribution over this set: $\rho : \Gamma \to [0,1]$

Drawing from this distribution yields *G*

Restrictions on the social network (r)

For each *x* in *D*, denote

$$\Gamma_j(x) = \text{subset of } \Gamma_j \text{ such that for each } X \in \Gamma_j(x),\ |X| = x$$

Restrict the distribution over r as follows:

$$\text{For each } i, \text{for each } j \in G_i, \Pr[G_j \in \Gamma_j(x) \mid G_i, \theta_i] = q(x)$$

$$\text{For each } i, \text{for each } j \notin G_i, \Pr[G_j \in \Gamma_j(x) \mid G_i, \theta_i] = \hat{q}(x)$$

Generalizes to posteriors conditional on degree
Admits generalized random graphs, standard models of "small world" networks

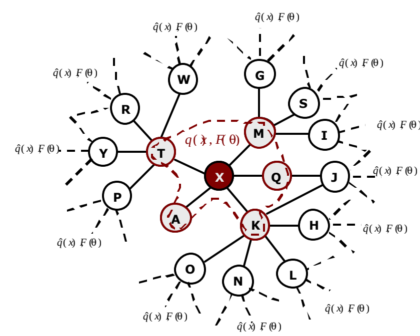
Sequence of the game

- Nature draws θ_i for each *i*, draws $G \in \Gamma$
- Each agent *i* observes their type
- Each agent *i* chooses either to adopt (a_i=*1*) or not (a_i=*0*)
- Payoffs are realized

Information

- After each agent realizes their neighbor set and type:
  - They know the exact structure of their local network
  - They have very little information about the structure of the rest of the network
    - Posterior $\hat{q}(x)$ on degree of non-neighbors
  - They have inexact (but better) information about the structure of the local networks they belong to
    - Posterior $q(x)$ on degree of neighbors
  - They know their θ, do not know anyone else's

Information



Equilibria

- Each symmetric Bayes-Nash equilibrium involves a threshold strategy:

$$s(d_i, \theta_i) = \begin{cases} 0, & \theta_i < \theta^*(d_i) \\ 1, & \theta_i \geq \theta^*(d_i) \end{cases}$$

with threshold $\theta^* = [\theta(1), \theta(2), \ldots, \theta(m)]$

- "No adoption" is always an equilibrium for pure network goods
- The equilibria can be Pareto ordered: $\Theta^* = \{\theta^A, \theta^B, \ldots\}$

$$\theta^A < \theta^B < \ldots$$

Main theory results

- The ordering of equilibria is based on the equilibrium probability of neighbor adoption

$$\lambda(\theta) = \sum_{x=1}^{m} q(x)[1 - F(\theta(x))]$$

- "Higher" equilibria strictly Pareto-dominate lower ones, and therefore, there is a best equilibrium, which has the highest value of $\lambda(\theta^*)$
- Each fulfilled expectations outcome with a local expectation λ of neighbor adoption has a corresponding Bayes-Nash equilibrium with $\lambda(\theta^*) = \lambda$
  - Coordinating adoption may be simpler if it is (a) local and (b) based on a simple parameter
- Greatest equilibrium is "weakly" coalition proof: establishes a basis for stability in the standard model

The structure of adoption networks

Consider a generalized random graph with degree distribution $p(x)$, and moment generating function (MGF)

$$\Phi_p(w) = \sum_{x=0}^{\infty} p(x) w^x$$

For identical θ, and for a threshold degree δ^*, the MGF of the degree distribution of the adoption network is

$$\Phi_\alpha(w) = \Phi_p[1 - \bar{Q}(\delta^*) + w\bar{Q}(\delta^*)]$$

where

$$\bar{Q}(x) = \Pr[d_j \geq x \mid j \in G_i] = \sum_{j=x}^{m} q(x)$$

Summary: Models of networks

Network effects in economics

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- Network effects are "global"
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Summary of results

- Simple way of modeling adoption of a technology with local network effects as a game of incomplete information between agents connected in an underlying social network
- This game has at least one (and generally many) symmetric Bayes-Nash equilibria in pure strategies
  - All equilibria involve generalized threshold strategies (a threshold degree associated with each value of the agent's "strength" of network effect)
  - These equilibria can be strictly Pareto ranked, based on a simple parameter: the probability a neighbor might adopt
  - One-to-one mapping between equilibria of the game and "fulfilled expectations equilibria" with local expectations

Summary of results

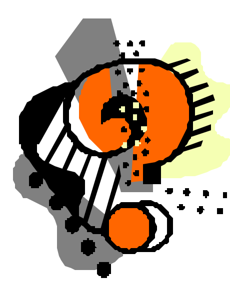
- A simple closed-form expression that describes the structure of an adoption network in terms of the structure of the social network (and vice versa)
- Some answers to other questions
  - Monopoly pricing is generally higher than a standard model that ignores network structure would predict
  - A monopolist always gives free versions to a fraction of their customers (and if possible, would target low-degree customers rather than highly connected customers)
  - The social optimality of universal access (or the optimality of mandated IT standards) relies on social/business networks not being too clustered.

Summary of related results

In a model of adoption with "boundedly rational" expectation formation, and bounded attention to changes in prices:

- For the corresponding model with unboundedly rational consumers: constant optimal price.
- This rational expectations equilibrium price is never a steady state of the optimal dynamic pricing policy
- When customers are myopic, for a range of forms of customer heterogeneity, the optimal price path is a target policy:
  - **Price at zero until a critical mass is reached ("bargains")**
  - **Set a steady state price, higher than the price predicted by the rational expectations model, after critical mass is reached ("ripoffs")**
- This result generalizes to
  - **Mixtures between myopic and unboundedly rational**
  - **Mixtures between myopic and "stubborn" (for at least one example)**

Questions and discussion?



http://oz.stern.nyu.edu/research/