Adoption games with network effects: a generalized random graph model
Arun Sundararajan
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Overview
- Network effects are often "local"
  - Communication technologies, business networks, online marketplaces...
- The structure of underlying social or business networks affects the adoption of network goods
  - An agent’s "local" network directly affects their value from adoption...
  - ...but so does the structure of the rest of the social network
  - Local networks are connected
  - One’s neighbors’ local networks affect their adoption decisions

Overview
- Agents in this kind of network generally have:
  - good (perfect) information about the structure of their own local network
  - some information about the structure of the other local networks they belong to (their neighbor’s local networks)
  - very little or no information about the exact structure of the rest of the social network
- Many useful probabilistic abstractions of networks (graphs) have been developed recently
  - Newman, Watts, Strogatz (generalized random graphs)
  - Watts and Strogatz (small-world models)
  - Price, Albert and Barabasi (preferential attachment models)

Overview
- Objectives
  - To model costly adoption of a product with local network effects (a model of demand with local network effects)
  - To apply this model to study a bunch of research questions
- Progress thus far
  - Modeled adoption game with a single product, pretty general agent characteristics and network structure
  - Studied (briefly) what the structure of the "adoption networks" look like
  - Answered (or answering) some simple questions: monopoly, monopoly with free samples, monopoly with an installed base, duopoly with identical products, differentiated duopoly

Snapshot of some results
- Adoption game has at least one (and generally many) symmetric Bayesian Nash equilibria
- All equilibria involve (generalized) threshold strategies
- Equilibria can be strictly (Pareto) ordered, based on a simple parameter (equilibrium probability of neighbor adoption)
- There is always a best equilibrium, which is "coalition proof"
- Each Bayesian Nash equilibrium corresponds to a "fulfilled expectations" equilibrium, and vice versa
- Adoption networks have some interesting structural properties
- Some answers to other questions
  - Monopoly pricing is generally higher than a standard model that ignores network structure would predict
  - A monopolist always wants to give free versions to a fraction of their customers (if targeted, to low-degree customers)
  - The only duopoly equilibrium that is ‘stable’ involves marginal cost pricing
Model

- Set of potential customers $K = \{1, 2, 3, \ldots, M\}$
- Single homogeneous network good that costs $c$
- Customers connected by an underlying social network (more on this in a couple of slides)
- Each customer has:
  - A neighbor set $N_k = \{N_{(k,1)}, N_{(k,2)}, \ldots, N_{(k,n)}\}$
  - A degree (number of neighbors) $n_k$
  - A type (index of valuation of product)
- Each customer makes an adoption choice $a_k \in \{0, 1\}$
- Value from adoption for customer $k$:
  $$\max \left\{ u\left( \sum_{j \neq k} a_j \theta_j \right) - c \right\}$$
- More generally: any situation with local externalities

Model

- Social network: instance of generalized random graph with degree distribution $p(x), x \in \{0, 1, 2, \ldots, m\}$
- How are these graphs constructed?
  $$\begin{align*}
    n_k = 2 & \quad \circ \quad \circ \\
    n_k = 3 & \quad \circ \quad \circ \quad \circ \\
    n_k = 5 & \quad \circ \quad \circ \quad \circ \quad \circ \\
    n_k = 2 & \quad \circ \quad \circ \\
    n_k = 2 & \quad \circ \quad \circ \\
    n_k = 5 & \quad \circ \quad \circ \quad \circ \\
    n_k = 5 & \quad \circ \quad \circ \quad \circ \quad \circ
  \end{align*}$$

Model

- Social network: instance of generalized random graph with degree distribution $p(x), x \in \{0, 1, 2, \ldots, m\}$
- How are these graphs constructed?

Model: Sequence of the game

- Nature creates the social network (according to the random graph algorithm), draws types for each agent
- Each agent $k$ observes their type, their neighbor set, and (therefore) their degree
- Each agent $k$ chooses either to adopt ($a_k = 1$) or not ($a_k = 0$)
- Payoffs are realized

Model: Information

- After each agent realizes their neighbor set and type:
  - They know the exact structure of their local network
  - They have very little information about the structure of the rest of the network
    - Posterior $p(x)$ on degree of non-neighbors
  - They have inexact (but better) information about the structure of the local networks they belong to
    - Posterior $q(x)$ on degree of neighbors
  - They know their type, do not know anyone else’s type
    - Posterior $F$ on all other agents
  - The results should hold for correlated degree, type
Model: Equilibria

- Each symmetric Bayesian Nash equilibrium involves a threshold strategy:
  \[ s(n, \theta) = \begin{cases} 
  0, & \theta < \theta^*(n) \\
  1, & \theta \geq \theta^*(n)
  \end{cases} \]

  with threshold \( \theta^* = [\theta(1), \theta(2), ..., \theta(m)] \)

- "No adoption" is always an equilibrium

- The equilibria can be ordered: \( \Theta^* = \{ \theta^d, \theta^g, ... \} \)

  \( \theta^d < \theta^g < ... \)

Model: Equilibria

- This ordering is based on the equilibrium probability of neighbor adoption:
  \[ r(\theta^d) = \sum_{n=1}^{m} q(n) (1 - F(\theta^d(n))) \]

- "Higher" equilibria strictly Pareto-dominate lower ones, and therefore, there is a best equilibrium, which has the highest value of \( r(\theta^d) \)

- Is coordination simpler if it is (a) local and (b) based on a simple parameter?

- Each "fulfilled expectations" outcome with expectation \( r \) has a corresponding Bayesian Nash equilibrium with \( r(\theta^d) = r \)

- The best equilibrium is the unique coalition-proof correlated equilibrium

Example: Complete social network

- \( p(M-1) = 1, p(n) = 0 \) for \( n < (M-1) \)

- Social network is complete graph

- This corresponds to a standard model

  - "Fulfilled expectations" equilibria with a continuum of types and customers always have an "outcome equivalent" Bayesian Nash equilibrium in an \( M \)-player adoption game with heterogeneous types

  - Perhaps the latter is a better choice, because it allows one to examine stability more closely

Example: Pure random graph

- \( F(1) = 1, F(0) = 0 \) otherwise

- Adoption is completely determined by structure of the social network:
  \[ s(n) = \begin{cases} 
  0, & n_k < n^* \\
  1, & n_k \geq n^*
  \end{cases} \]

- Structure of the "adoption network"

Structure of adoption networks

\[
\Phi_p(w) = \sum_{x=0}^{\infty} p(x) w^x : \text{moment-generating function of the degree distribution of the social network}
\]

\[
\Phi_a(w) = \sum_{x=0}^{\infty} a(x) w^x : \text{moment-generating function of the degree distribution of the adoption network}
\]

Then, for a pure random graph:

\[
\Phi_a(w) \equiv \Phi_p(1 - Q(\delta^*) + wQ(\delta^*))
\]

Summary

- Perhaps the latter is a better choice, because it allows one to examine stability more closely