Dynamic Pricing of Network Goods with Boundedly Rational Consumers

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Overview

- When is unbounded rationality a good approximation?
- Our (tentative) approach to answering this question is by studying a series of examples.
  - Choose a set of 'standard' economic models in which agents have unbounded rationality (the UR models).
  - Analyze a model of the same phenomenon in which agents have bounded rationality (the BR models).
  - Compare the "output" of the two models.
- Our first example: monopoly pricing for a network good.

A model of network goods

- A monopoly firm sells a homogeneous network good (a service, rather than a durable good).
- Unit mass of a continuum of consumers, indexed by their type \( \theta \in [0,1] \) drawn from a distribution with CDF \( F \).
- If the price in any period is \( p \), then a consumer of type \( \theta \) purchases the good in that period if
  \[ q_E(\theta) \geq p, \]
  where \( q_E(\theta) \) is the total demand expected by the consumer in that period.
- Variable cost equals zero.

A discrete-time formulation

- Suppose the firm varies its price at equally-spaced time intervals \( t = 0, h, 2h, 3h, \ldots \)
  \( h \) is the length of the time interval (more on this later).
- Sequence of events for a UR model
  - The firm announces its price \( p(t) \).
  - Each consumer forms an expectation of demand \( q_T(t) \).
  - A consumer of type \( \theta \) purchases if \( q_T(\theta) \geq p(t) \).
  - The realized demand is \( q(t) = 1 - F\left(\frac{p(t)}{q_T(t)}\right) \).
  - The firm’s profit in period \( t \) is \( pt - qt F(\theta - \theta) \).

Outcomes in a UR model

- Since consumers are unboundedly rational, they form rational demand expectations, which are fulfilled.
  \[ q(t) = 1 - F\left(\frac{p(t)}{q_T(t)}\right) \]
- The firm sets the same price \( p(t) \) in each period, and demand is constant across time.
- For instance, if \( F(\theta) = 0 \), then \( q^{UR}(t) = 1 - \frac{1}{\theta} \).
- Why the UR model seems implausible for this problem:
  - The extent of knowledge and computation that the model has consumers performing seems high (identifying other consumers’ preferences, forecasting demand based on these preferences,…)
  - The predictions of the model do not appear to be consistent with observed pricing and demand patterns.

Sequence of events in a BR model

- The firm announces its price \( p(t) \).
- Consumers who pay attention to \( p(t) \):
  - Determine some subset of past demand \( q(t-k), q(t-2k), \ldots \)
  - Form an expectation of demand \( q_T(t) \).
  - Make a decision based on the relative values of \( p(t) \) and \( q_T(t) \).
- Consumers who do not pay attention to \( p(t) \) continue doing what they were doing in period \( (t-h) \).
Modeling bounded “cognition”

- **Attention:**
  - If the length of the time interval is \( h \), then a fraction \( \lambda h \) of consumers of each type pay attention to \( p(t) \) in period \( t \), and make a decision.

- **Ability to forecast:**
  - Unboundedly rational: \( q_E(t) = q(t) \).
  - Myopic: \( q_E(t) = q(t-h) \).
  - Myopic and stubborn: \( q_E(t) = \gamma q(t-h) + (1-\gamma)q(t) \).

A continuous-time approximation

- If \( 0 \leq p(t) \leq q(t) \), and under the following BR model:
  - Bounded attention: If the length of the time interval is \( h \), then a fraction \( \lambda h \) of consumers of each type actually make a decision in period \( t \), and
  - Myopic forecasts: \( q_E(t) = q(t-h) \),
  then the time-rate of change in demand as \( t \to 0 \) is:

\[
q'(t) = \begin{cases} 
0, & q(t) = 0; \\
\lambda \left[ \left( 1 - \frac{p(t)}{q(t)} \right) - q(t) \right], & 0 < q(t) \leq 1, 0 \leq p(t) \leq q(t); \\
-\gamma, & 0 < q(t) \leq 1, p(t) > q(t).
\end{cases}
\]

- This law of motion remains unchanged for forecasts that are "more rational" than myopic (more on this later).

Summary of the firm’s problem

- Chooses the price trajectory \( p(t) \) that maximizes:
  \[
  \int_0^\infty e^{-rt} p(t)q(t) \, dt
  \]
  subject to the law of motion.

We can restrict our attention to stationary policies \( p(t) = q(q(t)) \).

The value of a policy \( \alpha \) at an initial state \( q \) is:

\[
V_{\alpha}(q) = \int_0^\infty e^{-rt} q(q(t))p(t) \, dt, \quad V_{\alpha}(0) = q.
\]

The value function at an initial state \( q \) is:

\[
V(q) = \sup_{\alpha} V_{\alpha}(q).
\]

A policy is optimal if its profit attains this supremum at every state \( q \).

Recall the UR model

- Under the UR model, demand in any period satisfies rational expectations:
  \[
  q = 1 - \frac{q}{F(\frac{q}{2})}.
  \]

For each \( q \), define \( P(q) \) implicitly as the largest solution of the above equation:

\[
P(q) = \max \{ p : q = 1 - F(\frac{p}{2}) \}.
\]

- (also the best "stay-where-you-are" price at \( q \)).

- Under the optimal rational-expectations equilibrium, demand solves:
  \[
  q^{UR} = \arg \max_q [qP(q)].
  \]

Results: Myopic consumers

1. The rational-expectations demand cannot be the steady state of an optimal price trajectory
   - \( q \) is a steady state for the optimal policy \( \alpha^* \) if 
     \[
     q(t) = q \text{ implies that } q(s) = q \text{ for all } s > t.
     \]
   - Theorem: The optimal rational expectations demand \( q^{UR} \) is not a steady state for the policy that this optimal for the BR model with myopic customers.

2. Solution to the optimal dynamic pricing problem
   - a "target policy."

   - When \( F(0) = 0 \) (uniform distribution of types), the firm’s optimal pricing policy is:
     \[
     \alpha^*(q) = \begin{cases} 
0, & q < \sigma^*; \\
P(q), & q = \sigma^*; \\
q, & q > \sigma^*.
\end{cases}
\]

where the optimal target \( \sigma^* = \frac{2\lambda}{3\lambda + r} < \frac{2}{3} = q^{UR} \).
Results: Myopic consumers
2. Solution to the optimal dynamic pricing problem
   - a "target policy."
   - Variation in optimal policy for
     - $F$ concave
     - $F$ convex

Myopic and stubborn consumers
- Attempt to see what happens when consumers are less rational than myopic.
- Consumers base their demand forecast on a weighted average of the myopic forecast and a shared stubborn forecast $\omega$.
  \[
  q_E(t) = \gamma q(t) - h + (1 - \gamma)\omega
  \]
  $\omega$: a fixed parameter.
  $\gamma = 1 \Rightarrow$ consumers are purely myopic.
  $\gamma = 0 \Rightarrow$ consumers are purely stubborn.

Myopic and stubborn consumers
- Law of motion:
  \[
  q(t) = \begin{cases} 
  0, & q(t) = 0 \\
  \lambda \left[ 1 - F \left( \frac{p(t)}{\lambda \sigma \omega} \right) - q(t) \right], & 0 < q(t) \leq 1, 0 \leq p(t) \leq \gamma q(t) + (1 - \gamma)\omega \\
  \infty, & 0 < q(t) \leq 1, p(t) > \gamma q(t) + (1 - \gamma)\omega
  \end{cases}
  \]

Myopic and stubborn consumers
- Preliminary results
  - The monopolist’s optimal price trajectory is generated by a target policy with target $\sigma(\gamma, \omega)$.

Myopic and stubborn consumers
- Preliminary results
  - $\sigma(\gamma, \omega)$ is strictly increasing in $\gamma$, and has the following values at its end points:
    \[
    \sigma(0, \omega) = \begin{cases} 
    \frac{\lambda}{2(2\lambda + \rho)}, & \text{concave network value function and uniform distribution of types.}
    \end{cases}
    \]

Concluding remarks
- Target policy more realistic than REE.
- Model with both myopic and UR customers.
- Concave and convex network value functions – e.g., concave network value function and uniform distribution of types.
- Competing network goods.
- Decisions based on local network structure.
- Adaptive expectations, noisy observation.