PROMOTIONAL CHAT ON THE INTERNET [*]

Dina Mayzlin

Yale School of Management

September 24, 2001

[*] This is a preliminary draft. Please do not quote without the author's permission. I am grateful to Duncan Simester, David Godes, Jose Silva, Jiwoong Shin, On Amir and especially Birger Wernerfelt for helpful comments on earlier drafts of this paper. I would also like to thank seminar participants at University of Washington, Carnegie Mellon, Purdue, UT Dallas, Washington University, Yale, University of Florida, Rochester, University of Toronto, Northwestern, Duke, Berkeley, Cornell, UCLA, and USC. Please address correspondence to: dmayzlin@mit.edu

PROMOTIONAL CHAT ON THE INTERNET

Chat rooms, recommendation sites, and customer review sections provide consumers with an opportunity to overcome geographic boundaries and to communicate based on mutual interests. However, marketers have incentives to anonymously supply promotional chat or reviews in order to influence the consumer's evaluation of their products. This, in turn, lowers the credibility of word of mouth transmitted online. We develop a game theoretic model where an incumbent and an entrant that are differentiated in quality compete for the same online market segment. The consumers are uncertain about the entrant's quality, whereas the firms know the value of their products. The consumers hear messages online that make them aware of the existence of the entrant as well as help them decide which product is superior. We find a unique equilibrium where online word of mouth is informative despite the promotional chat activity by competing firms. In this equilibrium, we find that firms spend more resources chatting up inferior products. We also find that promotional chat may be actually more beneficial to consumers than a system with no promotional chat. There are a number of extensions that we explore in this paper. Thus, we discuss how results change under different assumptions on the cost function of messaging and discuss price signaling in the context of online chat.

1. Introduction

In August of 1999, teenagers who frequented online bulletin boards of Britney Spears, a teen pop star, began to receive messages that recommended a new singer: Christina Aguilera. The authors of the messages frequently identified themselves by their first names only. Thus, Britney's fans had no means of distinguishing whether the messages they received came from other fans or from a marketing firm.

Some of the messages sent out did disclose that the authors were employees of Electric Artists, a promotional firm that specializes in online marketing. (The company's motto is "Shaping the Future of the Music on the Internet"). Electric Artists hired "posters" to surf various chat rooms and fan sites in order to generate online discussion and to provide information to potential fans. The campaign was ruled a success since Ms. Aguilera's album debuted at No. 1 on the charts and reached double platinum status. In early October of 1999, The Wall Street Journal devoted a front-page story titled "'Chatting' a Singer Up the Pop Charts" describing the various stages of the campaign.

A remarkable feature of this campaign was the means of communication used by the marketers: the Internet enabled the promoters to infiltrate and influence consumers' conversations. At first, this sounds like a very attractive strategy for marketers of many types of products. After all, in the past few years we have seen a proliferation of online communities: one community search engine (forumone.com) lists 310,000 web forums (which include "discussion forums, bulletin boards or message boards.") These forums host conversations between consumers with diverse topics. One reason behind Electric

Artists' success was the fact that consumers often offer unsolicited product recommendations online, which lent some credibility to chat about Christina. Following the success of Christina's promotion, Electric Artists has expanded its client list to include Tommy Hilfiger, the cellular provider Air Touch, YM, as well as Universal studios. Another Net promotion agency, M80 Interactive, employs similar marketing techniques. M80 was behind the marketing effort of the top-selling *NSYNC album, "No Strings Attached." (Advertising Age; Chicago; May 1, 2000).

The marketers' ability to disguise their promotion as consumer recommendations is made possible by the anonymity enjoyed by participants of online communities. To quote Nirav Tolia, Epinions' co-founder, "The problem with most user-generated content on the Web is that there is no transparency, no context." (LA Times, 12/03/99, A-1, "Everyone Is a Critic in Cyberspace.") Some sites, such as Epinions, try to provide transparency by having users rate each other's reviews. However, a determined reviewer can enhance her ratings by having her friends contribute positive comments. There is even a term for such practice: "feedback abuse." (See "Building Stronger Brands Through Online Communities," Gil McWilliam) Ultimately, our identities as well as our incentives are obscure in the virtual world. Thus manufacturers can easily listen to the conversations that take place between what they think are consumers as well as actively participate in these discussions.

On the other hand, we might question the viability of such marketing efforts in the face of consumer skepticism. Consumers' awareness of the existence of such anonymous promotion (what we will call from now on "promotional chat") could cause them to discount online recommendations. Moreover, we would expect that the incumbent rival would engage in similar promotion to defend her market share. This paper poses the question whether promotional chat is a viable strategy in the long run.

More formally, this paper poses the following three research questions. First, we investigate conditions under which word of mouth online remains informative to the consumers in the presence of promotional chat by rival firms. Do we expect that anonymity, an aspect of the Internet that makes promotion so attractive, would be the undoing of promotional chat? Second, we ask whether promotional chat is most valuable for a firm whose product is more appealing than the competitor's product or a firm whose product is less appealing. Note that in previous advertising models firms spend more resources promoting their winners, which guarantees that advertising is a credible signal of quality. On the other hand, we also observe online recommendations of inferior products and questionable remedies: a post on sci.med.prostate.cancer states, "New results from Shark Cartilage show how it has helped many people reduce and combat cancer." Third, we ask how consumer welfare is affected by anonymity and promotional chat compared to a context where promotional chat is not allowed.

We propose a game theoretic model where the incumbent and the entrant firms hold private information concerning the quality of their products. The firms send costly recommendations to the consumer in order to influence his inference on the relative quality of the competing products. Consumers who have tried both products earlier also post online recommendations. Thus, online discussions are a mixture of unbiased recommendations as well as promotional activity by interested parties where the consumer is not able to tell apart the advertising from unbiased content. The consumer makes an inference on the quality of the new product based on the recommendation she receives. The consumer's inference will be affected by her knowledge that the firms engage in promotional chat.

We find that if the costs of engaging in promotional chat are sufficiently high, online chat remains informative. Thus, the firms' promotional activity does not turn chat rooms into noise: consumers are still more likely to hear the truth. Second, we find that promotional chat is more effective for products of low quality: firms lie. Note that the latter is the opposite of the signaling literature result where firms find it more profitable to promote their winners. The first and second results taken together are surprising: despite the firms' incentives to invest more into promoting the less appealing products, the consumers find chat informative. Third, we find that under certain conditions consumers may actually benefit from promotional chat due to the fact that it increases awareness of the new product.

2. Literature Review

This paper relates to the existing literature on advertising and word of mouth communication. There is a rich literature in marketing on the sales response to advertising and the creation of an optimal advertising campaign. See, for example, Vidale and Wolfe (1957), Little and Lodish (1969), Sasieni (1971), Little (1979), Simon (1982), Mahajan and Muller (1986), and Feinberg (1992). These are aggregate models in the sense that the consumer’s decision is not modeled on an individual level, but the consumer’s actions are summarized as a response function to sales. Thus, this literature is not concerned with questions of credibility of communication; questions that are central to this paper.

Another related stream of literature is new product diffusion through word of mouth and advertising. See, for example, Dodson and Muller (1978) and Mahajan, Muller and Kerin (1984). These papers model consumers as divided into segments where the consumers are either unaware of the product, aware of the product but had not yet purchased it or who have already purchased the product. Once again, these models do not address the questions of credibility. Instead, they take a somewhat mechanistic view of consumers’ product choices: the flow between segments depends on the amount of advertising as well as word of mouth, which is in turn a function of relative sizes of the segments. Monahan (1984) addresses a similar problem using optimal control methodology.

In the economics literature, Ellison and Fudenberg (1995), Banerjee (1992) and Banerjee (1993) model the spread of information among consumers. McFadden and Train's (1996) model deals with consumers optimally choosing between trying a new product early on or waiting for others’ reviews. Vettas (1997) examines the optimal timing of a monopolist’s sales given word-of-mouth communication between consumers. Avery et. al. (1999) deals with the organization of a market for product evaluations. Similarly to the approach taken in this paper, the papers above model consumers as Bayesian updaters (with the exception of Ellison and Fudenberg (1995) where more simple decision rules are used). In the models above, a consumer imperfectly learns from others' experience since there is heterogeneity in preferences or, in the case of Banerjee's model, uncertainty whether previous consumers acted on new information or "herded." However, none of the papers above model the firms' incentives to directly manipulate word of mouth.

Finally, there is a rich stream of literature that deals with the type of information that is conveyed by advertising. Dixit and Norman (1984) and Stegeman (1991) deal with the provision of hard information, namely, prices. Dixit et. al. (1984) finds that firms advertise excessively with respect to the social optimum, while Stegeman (1991) finds the opposite result : competitive firms usually underadvertise. Nelson (1974) and Kihlstrom and Riordan (1984) deal with the provision of soft information: the signaling value of advertising. Thus, in both of the models above advertising is a credible signal of quality. Advertising is a credible signal either due to the high quality firm's ability to recover the costs in repeat purchases or the high quality firm's lower costs of production. Note that in our model the costs of production do not correlate with quality and there is no possibility of a repeat purchase since we consider a one-period model. Unlike the models above, Horstmann and Moorthy (2000) show that advertising does not need to be monotone increasing in quality. They develop a model of service that includes a technological relationship between quality and capacity where high quality services cannot be provided by large capacity firms. Thus, in the presence of capacity constraints, the high quality firms derive low value of advertising. This paper shows that even in the absence of capacity constraints, promotion need not be increasing in quality.

3. Basic Model

The model in this section closely follows the example presented in the introduction. Here we present the set-up and the results of the basic model. In Section 3, we discuss an extension that deals with comparing consumer welfare across different game forms. In the last section, we discuss the results and present ideas for future research.

Let us first present a general overview of the model. There are two firms, an incumbent and an entrant, one risk-neutral uninformed consumer, and a segment of informed consumers. The firms offer substitute products of different quality. The firms observe which is the better product, but the uninformed consumer only observes the quality of the product offered by the incumbent firm. The segment of informed consumers have tried both products and thus also observe which is the superior product. Thus, we assume that the firms, through market research, perhaps, or experience, possess better knowledge of the industry than do the consumers who have not purchased the new product. (In the music industry, for example, audience tests can be conducted where a consumer is paid to rate snippets of various songs). More formally, we assume that the uninformed consumer expects a sure payoff of V^B from the incumbent's product (B) and an uncertain payoff of either $\{ V^C = V^C_H > V^B \}$, which we call State 1, or $\{ V^C = 0 \}$, which we call State 2, from the entrant's product (C). Thus, in State 1, C is the more appealing product, whereas in State 2, C is the less appealing product.

Moreover, we assume that the uninformed consumer only becomes aware of the entrant through the promotional chat. That is, the consumer has a prior belief on the expected quality of an entrant, but cannot buy the entrant's product unless he hears a message mentioning the entrant's name. For instance, the consumer knows that 50% of new artists are appealing, but must learn from the chat the new artist's name before buying her CD. Thus, promotional chat serves two functions: awareness as well as recommendation on product choice.

Next, let us turn to the messaging that takes place online. There are three possible types of messages: messages that claim that the entrant's product is better than the incumbent's product, messages that claim that the incumbent's product is better than the entrant's product, and messages that pertain to the incumbent only. Alternatively, we can interpret these messages as positive word of mouth concerning the entrant, negative word of mouth concerning the entrant, or messages that do not mention the entrant. We model anonymity by assuming that consumers can't see the source of the message that they receive. This is a crucial assumption, and we later discuss how the presence of anonymity affects the results.

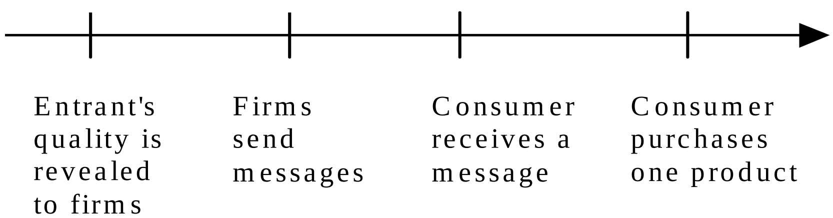
We also assume that the consumer observes only one message and makes an inference on the relative quality of the products based on that one message as well as his knowledge of the firms' actions in equilibrium. Thus, the consumer is unable to visit all the chat rooms, and observes a small subset of all the messages that are sent out. An alternative way to model this phenomenon would be to assume that the consumer receives a sample of total messages and updates his beliefs based on the ratio of positive messages received about the entrant. This alternative modeling technique would retain the flavor of the "one message" assumption: the more messages a firm sends, the more likely it is to convince the consumer of the high appeal of its product.

Let us next turn to the messages sent by the consumers. We assume that the informed segment of consumers sends N^U reviews that reveal the truth about the relative quality of the two products. We can think of this informed segment as consumers who have early knowledge on the quality of the entrant. For example, these are the teenagers who hear Christina's single and go online to talk about her. We also assume that the informed consumers send messages that contain information about B only. For example, these messages may discuss Britney's outfit at an awards ceremony. We assume that there are N^0 of these irrelevant messages. Note that even though these messages are irrelevant from the perspective of product comparison, the participants may still enjoy sending as well as receiving these messages. We do not model the incentives of the informed unbiased consumers to post online. Instead, we implicitly assume that the posters are motivated by altruism, an assumption that is consistent with previous word of mouth literature. We do examine how the magnitude of these parameters affects our results.

On the firm's side, we assume that there are convex costs of messaging. This would model a situation where messages need to be personalized or it becomes more difficult to create an additional message since it has to visibly differ from the messages that come before it. We also discuss a variant of the model with linear costs. We focus on equilibria with price pooling and show that, in fact, there does not exist a price signaling equilibrium.

The equilibrium concept used in the model is Bayesian Nash Equilibrium. Thus, the firms condition their actions on the State of the world (the quality of the entrant), and the consumer tries to infer the State of the world based on the message he receives. In addition, the consumer updates his prior taking into account the firms' strategies in both States of the world. Thus, both the consumer and the firms are fully strategic. We investigate the firms' actions, as well as the "informativeness" of the system to the consumer. Figure 1 presents the sequence of events of the game.

Figure 1



3a. Firms' Problem

To solve for the equilibrium, we need to specify the number of messages that the two firms will choose to send in the two states of the world as well as the inferences that the consumer makes. Let us first turn to the firms' problem. The strategy space available to firms is the number of messages that they send praising their product. As a reminder, we refer to incumbent as B(ritney) and to entrant as C(hristina). Also, let {State 1} be the state of the world where the entrant's product is superior, whereas {State 2} is the state of the world where the incumbent's product is superior. (Thus, N_1^B is the number of messages sent by the incumbent in State 1; N_2^B is the number of messages sent by the incumbent in State 2; N_1^C and N_2^C are messages sent by the entrant; N^U is the number of unbiased messages praising the superior product; and N^0 is the number of messages that are irrelevant to product comparison).

In the model, a consumer receives one message only. The three possible types of messages that can be received are: $\{\mu^{B>C}, \mu^{C>B}, \mu^{B}\}$ where $\mu^{B>C}$ stands for a message praising B over C, $\mu^{C>B}$ stands for a message praising C over B, and μ^{B} stands for a message that is irrelevant for product comparison. Note that there will be $N^U + N_1^C$ messages praising C in State 1 since there will be N^U truthful unbiased messages and N_1^C messages sent by firm C. Similarly, there will be $N^U + N_2^B$ messages praising B in State 2. We assume that the consumer picks a message at random from the existing pool of messages. Thus, the probability that a consumer observes $\mu^{C>B}$ is the ratio of all messages praising C over the total number of messages. Hence, the probabilities that each type of message will be received by the consumer are summarized in Table 1 below.

Table 1: Probability the consumer receives a particular message

Type of Message	State 1	State 2
$\mu^{B>C}$	$\dfrac{N_1^B}{N_1^B + N_1^C + N^U + N^0}$	$\dfrac{N_2^B + N^U}{N_2^B + N_2^C + N^U + N^0}$
$\mu^{C>B}$	$\dfrac{N_1^C + N^U}{N_1^B + N_1^C + N^U + N^0}$	$\dfrac{N_2^C}{N_2^B + N_2^C + N^U + N^0}$
μ^{B}	$\dfrac{N^0}{N_1^B + N_1^C + N^U + N^0}$	$\dfrac{N^0}{N_2^B + N_2^C + N^U + N^0}$

From the firm's perspective, each event above can result in two possible actions by the consumer: either the consumer buys or does not buy the firm's product (for a profit of either P or 0). Let $\Pi^C_{[C>B]}$ stand for the profit that C derives following the consumer receiving a message $\mu^{C>B}$. (Likewise for all other messages as well as for Firm B). Note that we assume that $\Pi^{B}_{[B]}$ = P -- the consumer buys B if he does not become aware of the product's existence through the chat room. Thus, if he does not hear anything about the entrant (either negative or positive), he makes the default purchase: the incumbent's product. Also note that we assume that $V^B > P$, i.e. that the consumer derives positive value from the incumbent's product. We then can write out Christina's profit function in State 1 below:

$$\frac{N_1^C + N^U}{N_1^C + N_1^B + N^U + N^0}\Pi^C_{[C>B]} + \frac{N_1^B}{N_1^C + N_1^B + N^U + N^0}\Pi^C_{[B>C]} + \frac{N^0}{N_1^C + N_1^B + N^U + N^0}\Pi^C_{[B]} - a\frac{(N_1^C)^2}{2} \qquad \mathbf{(1)}$$

Consistent with the convexity assumption, we assume that the cost of sending messages is quadratic in the number of messages sent. We later discuss the results under the assumption of linear costs. We also include a parameter *a* on the cost function even though later on it will be more convenient to divide out the equation by *a*, and to talk about the profit/ cost ratio.

Since there are two firms and two states of the world, there are altogether four simultaneous maximization equations. We list the maximizations for State 1 below, while the whole system is listed in the Appendix.

1) For entrant (C) in State 1:

$$\max_{N_1^C} \frac{N_1^C + N^U}{N_1^C + N_1^B + N^U + N^0}\Pi^C_{[C>B]} + \frac{N_1^B}{N_1^C + N_1^B + N^U + N^0}\Pi^C_{[B>C]} + \frac{N^0}{N_1^C + N_1^B + N^U + N^0}\Pi^C_{[B]} - a\frac{(N_1^C)^2}{2} \qquad \mathbf{(2)}$$
$$\text{s.t.}\, N_1^C \geq 0$$

2) For incumbent (B) in State 1:

$$\max_{N_1^B} \frac{N_1^C + N^U}{N_1^C + N_1^B + N^U + N^0}\Pi^B_{[C>B]} + \frac{N_1^B}{N_1^C + N_1^B + N^U + N^0}\Pi^B_{[B>C]} + \frac{N^0}{N_1^C + N_1^B + N^U + N^0}\Pi^B_{[B]} - a\frac{(N_1^B)^2}{2} \quad \textbf{(3)}$$
$$\text{s.t.} N_1^B \geq 0$$

For example, consider an equilibrium where the consumer buys C if he receives the message $\mu^{C>B}$, but buys B if he receives $\mu^{B>C}$ or μ^{B}. Since the price of the product is P, the firms' profit functions look like:

Entrant (C) in State 1: $\frac{N_1^C + N^U}{N_1^C + N_1^B + N^U + N^0}\frac{P}{a} - \frac{(N_1^C)^2}{2}$

Incumbent (B) in State 1: $\frac{N_1^B + N^0}{N_1^C + N_1^B + N^U + N^0}\frac{P}{a} - \frac{(N_1^B)^2}{2}$

3b. Consumer's Problem

Here the consumer is trying to infer which product will deliver higher value: B or C. Thus, his strategy space is a decision on a purchase of one of the products based on the message received. Note that since in the model the two goods are substitutes, the consumer never chooses to buy both products.

If the consumer does not become aware of C, he buys B as default. Otherwise, the consumer updates his priors on the quality of the entrant, conditioning on the message received. His prior probability on the entrant delivering more value (State 1) is P(s1). Let $\theta(\{\text{message}\}) = P(\text{state1} | \{\text{message}\})$ -- consumer's posterior following a message. The notation is the following: $P(\mu^{C>B} | s1)$ is the likelihood that $\mu^{C>B}$ message would be received in State 1. We next apply the Bayes' Rule to derive the updating of the consumer's beliefs:

$$\theta(\mu^{C>B}) = \frac{P(\mu^{C>B} \mid s1)P(s1)}{P(\mu^{C>B} \mid s1)P(s1) + P(\mu^{C>B} \mid s2)P(s2)} \quad \textbf{(4)}$$

$$\theta(\mu^{B>C}) = \frac{P(\mu^{B>C} \mid s1)P(s1)}{P(\mu^{B>C} \mid s1)P(s1) + P(\mu^{B>C} \mid s2)P(s2)} \quad \textbf{(5)}$$

where the probabilities of receiving each message in the two states of the world are summarized in Table 1 and presented once again below:

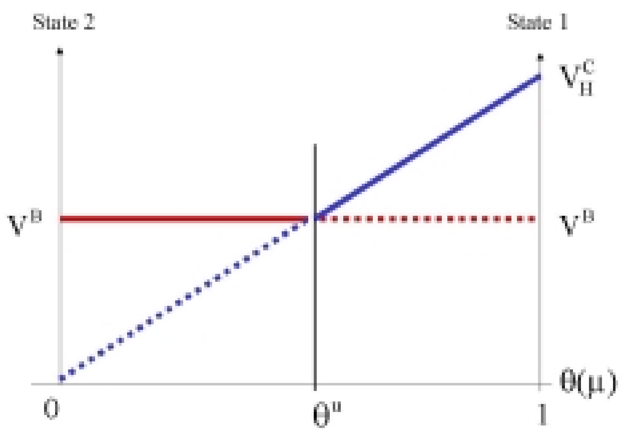
$$P(\mu^{C>B} \mid s1) = \frac{N_1^C + N^U}{N_1^B + N_1^C + N^U + N^0}\,;\ P(\mu^{C>B} \mid s2) = \frac{N_2^C}{N_2^B + N_2^C + N^U + N^0}$$

$$P(\mu^{B>C} \mid s1) = \frac{N_1^B}{N_1^B + N_1^C + N^U + N^0}\,;\ P(\mu^{C>B} \mid s2) = \frac{N_2^B + N^U}{N_2^B + N_2^C + N^U + N^0} \quad \textbf{(6)}$$

Thus, we can see that the consumer bases his decision taking into account the firms' optimal strategies in both states of the world. Note that he does not observe the firms' actions exactly, but instead calculates the equilibrium strategy.

Next we consider the relationship between the consumer's beliefs and his decision to purchase either C or B. Assuming risk-neutrality, Figure 2 below represents the decision that the consumer faces. The horizontal line represents the certain value from buying B. The positively sloped line represents the expected value of purchasing the entrant's product as a function of the posterior probability. The consumer maximizes his expected payoff by choosing the upper envelope of the two lines. If the consumer's posterior on State 1 is high enough ($\theta(\mu) > \theta^U$), the consumer would rather go ahead and purchase C. If, on the other hand, the posterior low enough ($\theta(\mu) \le \theta^U$), the consumer chooses to purchase B. Note that $\theta^u = \dfrac{V^B}{V_H^C}$.

Figure 2: Consumer's Optimal Value as a Function of the Posterior Belief



For example, what are the conditions that are required to obtain an equilibrium where the users follow the recommendations that they receive online? (That is, when will the consumer buy B when he receives $\mu^{B>C}$ and buy C when he receives $\mu^{C>B}$?) This will occur if $\theta(\mu^{C>B}) \geq \frac{V^B}{V_H^C}$ and $\theta(\mu^{B>C}) < \frac{V^B}{V_H^C}$. After we make the appropriate substitutions, we see that this is equivalent to the expressions below:

$$\frac{P(\mu^{C>B}\,|\,s1)}{P(\mu^{C>B}\,|\,s2)} \geq \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)} \text{ and } \frac{P(\mu^{B>C}\,|\,s1)}{P(\mu^{B>C}\,|\,s2)} < \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)} \qquad \textbf{(7)}$$

The above conditions ensure that the signals received are informative: the consumer's beliefs change enough to change consumer purchase decision.

3c. Bayesian Equilibrium

Putting together the firms' and the consumer’s problem together, we look for pure strategy equilibria where the consumer’s beliefs and firms' actions are mutually consistent. Once again, the equilibrium consists of the firms' decisions on how many messages to send contingent on the state of the world as well as on the consumer's decision which product to buy contingent on the message he receives.

Let us first consider the consumer’s problem. He has four possible strategies available to him. This is due to the fact that he may receive two possible messages and can choose whether to buy or not to buy the entrant following each message, where the decision not to buy the entrant’s product is equivalent to a decision to buy the incumbent’s product. By assumption, if the consumer receives an irrelevant message, his default purchase is the incumbent's product. Once the consumer’s decision rule is fixed, we can derive the optimal messaging policies of the two firms. Finally, we have to check that the consumer’s decision is optimal, given the firms’ strategies. We find that a unique pure strategy equilibrium exists in a region where costs are above a certain cutoff. In this equilibrium, we find that the consumers will follow the online recommendations, but that the firms have an incentive to lie.

Proposition 1:

Let us define $\rho = P/a$ (a ratio of profit to cost). For the set of parameters $\{P(s1), V_H^C, V^B, N^U, N^0\}$ (where $0 < P(s1) < 1$) there exists $\hat{\rho}$ such that for all $\rho \le \hat{\rho}$ there exists a unique pure strategy Bayesian Equilibrium to the problem above. (No pure strategy equilibria exist in the region $\rho > \hat{\rho}$).

1) In this equilibrium, the consumer buys C if he receives $\mu^{C>B}$ and buys B if he receives $\mu^{B>C}$: recommendations online are informative.

2) The resulting promotional intensities are such that $N_1^C < N_2^C, N_1^B > N_2^B$: firms promote more heavily in the state of the world when their product is inferior.

3) The firm expects to make higher profits in the state of the world where its product is superior. See Appendix for the proof.

We next discuss and graphically illustrate the intuition behind the results. Let us fix the consumer beliefs to be such that he buys C if and only if he hears $\mu^{C>B}$ and consider C's incentives to post messages. We also assume for now that the total number of messages of type $\mu^{B>C}$ is fixed across the two states of the world: $N_1^B = N_2^B + N^U \equiv N^B$ 1, and we define an additional variable: $T_i^C \equiv$ the total number of messages of type $\mu^{C>B}$ in state *i* ($T_1^C = N_1^C + N^U; T_2^C = N_2^C$).

Next, consider C's benefit and marginal benefit of messaging in State 1:

$$\text{Benefit}\,|\,s1 = P(\mu^{C>B}\,|\,s1)\rho = \frac{N_1^C + N^U}{N_1^B + N_1^C + N^U + N^0}\rho \equiv \frac{T_1^C}{N^B + T_1^C + N^0}\rho\,;$$

$$\frac{\partial \text{Benefit}|s1}{\partial N_1^C} = \frac{\partial \text{Benefit}}{\partial T_1^C}\frac{\partial T_1^C}{\partial N_1^C} = \frac{\partial \text{Benefit}}{\partial T_1^C}(1) = \frac{N^B + N^0}{(N^B + T_1^C + N^0)^2}\rho\,.$$

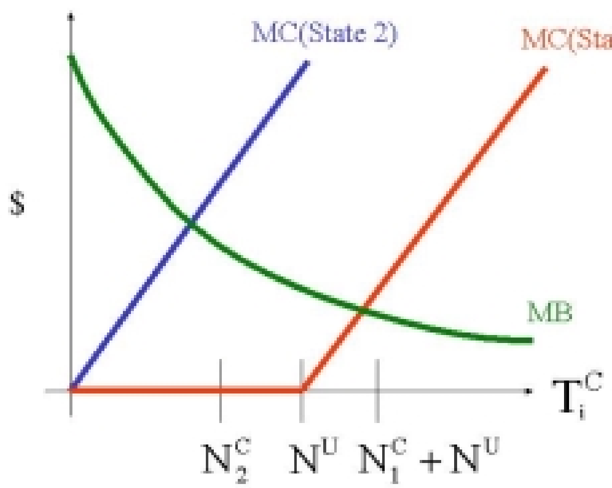
Similarly, in State 2,

$$\text{Benefit}\,|\,s2 = P(\mu^{C>B}\,|\,s1)\rho \equiv \frac{T_2^C}{(N^B + T_2^C + N^0)}\rho; \frac{\partial \text{Benefit}\,|\,s2}{\partial N_2^C} = \frac{N^B + N^0}{(N^B + T_2^C + N^0)^2}\rho \quad \textbf{(9)}$$

We can see that the marginal benefit (change in probability) is decreasing in the total number of $\mu^{C>B}$ messages sent. Note that in the example above the functional form of the marginal benefit as a function of T_i^C is the same for both states of the world.

Next, we turn to the marginal costs that C faces as a function of T_i^C. In State 1, C faces a marginal cost of 0 for N^U messages, and a linear marginal cost for each additional message. On the other hand, in State 2, C faces a linear marginal cost for all messages. The intersection of the marginal cost and marginal benefit curves determines the firm's strategy in both states of the world. Refer to the graph below for an illustration:

Figure 3: MB/MC Trade-Off



---

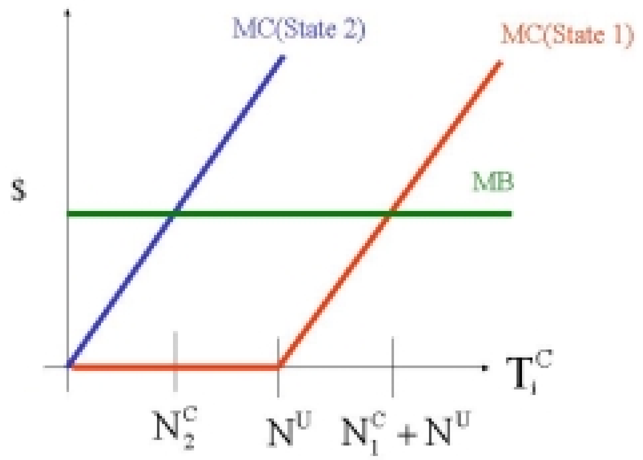
[1] Note that in equilibrium, $N_1^B < N_2^B + N^U$. In our illustration, we concentrate on the role of the cost structure. As will be shown below, this assumption allows us to fix the functional form of the marginal benefit across the two states of the world.

Thus, we can see that there will be more $\mu^{C>B}$ messages in the state of the world where C is superior: $N_1^C + N^U > N_2^C$. This is the informativeness result. That is, consumers on average should believe the recommendations that they hear. On the other hand, we see that $N_1^C < N_2^C$, i.e. the firms spend more resources promoting their losers.

There are three aspects of the model that are driving the results: 1) the declining marginal benefit of messaging 2) the existence of unbiased reviews, and 3) the convexity of the costs. We consider these three aspects separately.

Let us first illustrate that the declining marginal benefit of messaging is crucial for the result that firms invest more heavily in their losers. Along with unbiased reviews, this ensures that the superior firm's marginal benefit of messaging is lower than the inferior firm's. Below, we re-draw Figure 3 with a constant marginal benefit curve. We can see that here $N_1^C = N_2^C$ (with a linear marginal cost curve and a constant marginal benefit curve).

Figure 4: Constant MB



Of course, it is the micro model of consumer's drawing a message from a "bucket" of messages that results in the concavity of benefit. The assumption is meant to capture the reality that the more messages a firm sends, the more likely is a consumer to receive the promotional message.

Next, we turn to the assumption that there are unbiased truthful reviews out there. Note that this assumption is critical for obtaining the model's results. The difference between $N_1^C + N^U$ and N_2^C decreases as N^U decreases. Thus, no informative equilibrium exists as N^U goes to zero. How reasonable is this assumption? One possible criticism is that here the incentives of the unbiased reviewers are not modeled. This is a possible extension of the model. Moreover, it is possible that the number of unbiased reviews differs across categories. High involvement categories (such as movies) are likely to have more reviewing, while low involvement categories (coffee) are likely to have less reviewing. We can make predictions on the level of promotional activity based on the involvement of the category. We argue that this is a reasonable assumption due to the fact that people differ in their patience level (discount factor). Thus, some consumers are likely to quickly search out and try new products, while others are willing to wait and see how others react.

One of the attractive features of anonymous promotion on the Internet is the cost effectiveness of the campaign online compared to a similar campaign offline. Thus, the posters do not have to travel great distances to reach the consumers and can promote to an audience that is predisposed to buy a product in the category since the chat rooms are topic-based. Moreover, with the introduction of "chat bots" (programs that automatically generate chat) we might think that the costs of such a promotion are linear as opposed to convex. We thus consider how our results change if we assume that the costs are linear: each message costs a fixed amount, *a*. Once again, different values of *a* lead to different equilibria. In the Appendix, we formally define the different equilibria that are obtained in the various regions of the parameter space. In this section, we provide the intuition behind the results.

Proposition 2:
Consider the problem above with linear costs, where the cost per message is *a*. Let us similarly define $\rho = \frac{P}{a}$. We find the following:

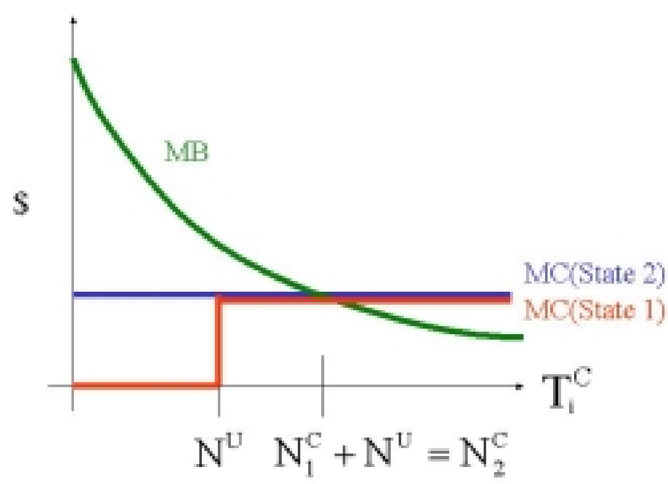
a) When $N^U + N^0 < \frac{\rho}{4}$ (relatively low costs), there is no informative pure strategy equilibrium.

b) When $\frac{\rho}{4} < N^U + N^0 < \rho$ (intermediate cost range), there are potentially informative pure strategy equilibria in the sense that $N_1^C + N^U > N_2^C, N_2^B + N^U > N_1^B$ and where $N_1^C < N_2^C, N_2^B < N_1^B$ (firms lie).

c) When $N^U + N^0 \geq \rho$ (high cost range), the firms choose not to promote their products, and the only messaging is done by the unbiased source.

See Appendix for the formal proof. Instead, we turn to the intuition behind the results in this section. Essentially, there are three possible cost levels: high, intermediate, and low. If the costs are low, there is no informative equilibrium (see Figure 5 below). We can see that the total number of $\mu^{C>B}$ messages is equal across the two states of the world. Thus, a message praising the product will not change the consumer's priors on the quality of the product, which renders the messages uninformative.

Figure 5: Low Linear Costs



However, if the costs are not too low, we run into corner solutions where either one or both of the firms choose not to expend any effort into promotion. Interestingly, these are informative equilibria. Figure 6 illustrates the situation where the costs are at an intermediate level. The superior firm chooses not to send any messages beyond N^U, whereas the inferior firm sends fewer than N^U messages. This is informative since a positive recommendation is an indicator that the product is of high quality. We can see once again that that firms lie (a firm spends more resources on its inferior products). Note that as we raise the costs even further (see Figure 7 below), we see that the firm chooses not to spend anything on promotion, regardless of the quality of their product. Here, no one promotes her product. All the recommendations a consumer sees online are sent by the unbiased source.

Figure 6: Intermediate Linear Costs

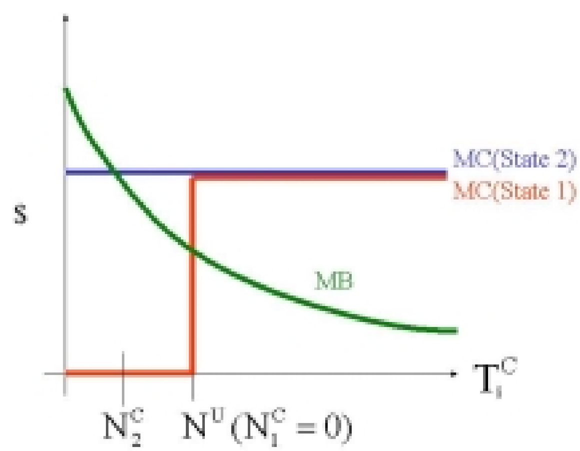
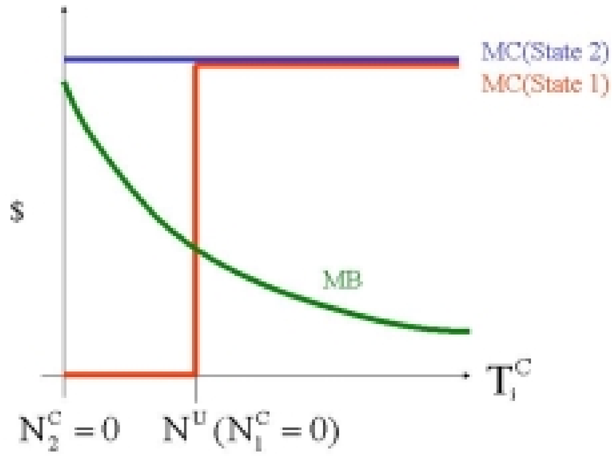
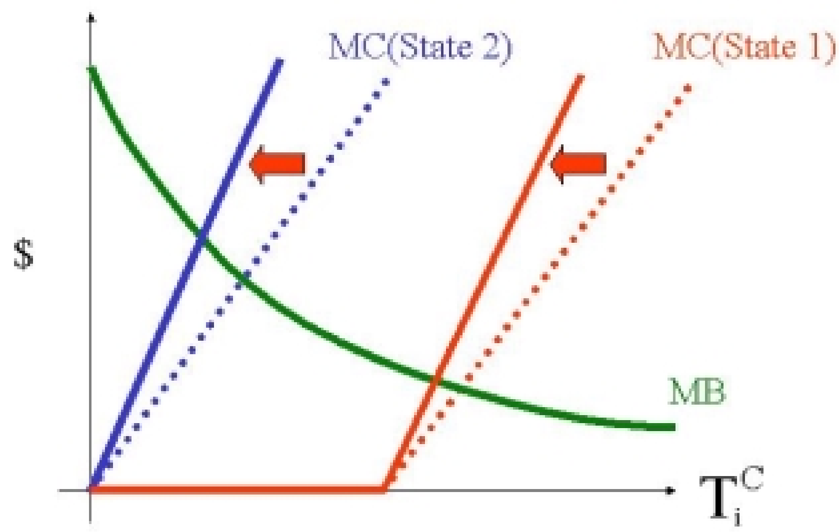


Figure 7: High Linear Cost



Note that with convex costs, we do not observe the discontinuities that we obtain with linear costs. Thus, firms never spend zero effort on promotion, no matter how high the cost parameter is. We again can demonstrate this graphically in Figure 8. Let us increase the *a* parameter. This rotates the marginal cost line upwards. This in turn decreases the number of messages sent by the firms, but will never decrease that number to 0 as long as $a < \infty$.

Figure 8: Increase in Convex Costs



Lastly, we contrast the finding of this paper with the findings of the advertising as signaling literature. As we mentioned, in that literature advertising provides soft information to the consumer. In that literature, the high quality firm can use advertising as a credible signal either due to differential costs or due to repeat purchase effects. That is, since the satisfied consumers will promote the high quality product to their friends, the firm with a better product benefits more from higher sales in the first period. Note that in our one-period, same cost model, we get the opposite result: the high quality firm views word of mouth as a substitute to advertising and advertises less than the low quality firm.

4. Consumer Welfare & Regulation

Next, consider to the issue of consumer welfare. In order to analyze how promotional chat and anonymity affect consumer welfare, we compare consumer welfare across two different game forms. We compare consumer welfare in the basic model against a system where all advertising is banned.

Proposition 3:

We find that promotional chat may benefit the consumer compared to a system with no advertising in the following two scenarios:

1) If the prior, P(s1), on the entrant being of high quality is high
2) If the system has a lot of irrelevant messages relative to unbiased consumers

Specifically, the consumer benefits from promotional chat if the following condition holds:

$$\frac{P(\mu^{C>B} \mid s1) - \dfrac{N^U}{N^U + N^0}}{P(\mu^{C>B} \mid s2)} \geq \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)}.$$ This simplifies to the following expression:

$$\frac{N^0 - N^U}{N^0 + N^U} \geq \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)}$$

See Appendix for proof.

Note that the result above is very intuitive. Thus, promotional chat is a good instrument for increasing awareness of the entrant. The consumer benefits from increased awareness only in the state of the world where the entrant is superior. (In the other state of the world the consumer follows the recommendation and is deceived). Hence, the prior on the entrant being superior is an important factor. On the other hand, the system of recommendations with no advertising sends out perfectly informative signals. However, the probability that such a signal is heard at all depends on the level of noise (number of irrelevant messages) relative to the level of unbiased messages. Thus, if noise is high relative to the number of unbiased messages, $\frac{N^0 - N^U}{N^0 + N^U}$ is high, the consumer is actually better off under a system where some advertising is allowed as opposed to a system with no advertising.

5. Extensions

In this section, we explore the robustness of the results in the main model by altering the specification of the model. In Section 5a, we examine several alternative specifications of the model. We consider a different process of message propagation that we call "seeding." We also consider a specification where the consumer can receive multiple messages. In Section 5b, we turn to price signaling, and in Section 5c, we discuss a mixing equilibrium.

5a. Alternative Specifications

For the purposes of this section only, we consider symmetric specifications only. (The precise definition of a symmetric specification is provided below). That is, in our original specification, an irrelevant message favored the incumbent due to the awareness assumption. Here we assume that there are no irrelevant messages present ($N^0 = 0$). We do this in order to explore how the asymmetry affects the results and also in order to simplify some of the later calculations. All other aspects of the model, including quadratic costs, remain the same.

To simplify the calculations, we define a few new variables. Following the notation above, we define $T_i^C \equiv$ the total number of messages of type $\mu^{C>B}$ in state i, and $T_j^B \equiv$

the total number of messages of type $\mu^{B>C}$ in state j. Below we summarize the probabilities that the consumer will receive either of the two messages in the two states of the world:

$$P(\mu^{C>B} \mid s1) \equiv f(T_1^C, T_1^B) = \frac{T_1^C}{T_1^C + T_1^B} \;;\; P(\mu^{B>C} \mid s1) \equiv 1 - f(T_1^C, T_1^B) = \frac{T_1^B}{T_1^C + T_1^B};$$

$$P(\mu^{C>B} \mid s2) \equiv f(T_2^C, T_2^B) = \frac{T_2^C}{T_2^C + T_2^B} \;;\; P(\mu^{B>C} \mid s2) \equiv 1 - f(T_2^C, T_2^B) = \frac{T_2^B}{T_2^C + T_2^B}; \quad \textbf{(10)}$$

Note that the probability function is symmetric since $f(T^C, T^B) = 1 - f(T^B, T^C)$.

Next, let us consider the issue of the asymmetry in the messaging between a firm with a superior product and a firm with an inferior product. Specifically, let us investigate how a firm's effort relates to the total number of messages that end up circulating online. In the main model, we assume that $T_1^C = N_1^C + N^U$, $T_2^C = N_2^C$, $T_1^B = N_1^B$, and $T_2^B = N_2^B + N^U$. Another reasonable specification is to assume that in the state of the world where the product happens to be superior there is some "seeding" of information. The term "seeding" was suggested by Ken Krasner, the CEO of Electric Artists, in a private conversation. In this view of promotional chat, the information supplied by the firm is multiplied and propagated by the informed consumers. For example, this process can be achieved if the informed consumer forwards to the uninformed consumer the firm's messages praising the superior singer in addition to generating messages on his own.

Mathematically "seeding" yields a specification $T_1^C = \kappa N_1^C + N^U$, $T_2^C = N_2^C$, $T_1^B = N_1^B$, and $T_2^B = \kappa N_2^B + N^U$ where $\kappa > 1$.[2]

With this specification once again we get results that are very similar to the main model. (See Appendix for a complete proof). That is, there exists $\hat{\rho}$ such that for all $\rho \le \hat{\rho}$ we obtain a unique informative pure strategy equilibrium where $T_1^C > T_2^C$ and $T_1^B < T_2^B$. In addition, the result that firms lie, $N_1^C < N_2^C$ and $N_1^B > N_2^B$, holds as long as $N^U > 0$. (We do, however, find that as k increases, $N_2^C - N_1^C$ and $N_1^B - N_2^B$ decrease). Similarly to the result obtained in the basic model, as N^U decreases, $N_2^C - N_1^C$ and $N_1^B - N_2^B$ decrease. Once again, the existence of independent word of mouth (unrelated to firm's actions) is essential for the result that firms lie.

Next, we explore a different specification for the probability function, $f(T^C, T^B)$. Specifically, recall that in the main model we make an assumption that the uninformed consumer receives one message only. Let us relax that assumption, and instead, assume that the consumer can receive multiple messages praising B and C. Once again, for simplicity we assume that the function is symmetric. That is, there are no irrelevant messages.

[2] Note that when $\kappa = 1$, we have the basic model where $N^0 = 0$.

In addition, let $\tilde{X}(\mu^{C>B})$ be the number of messages praising C and let $\tilde{X}(\mu^{B>C})$ be the number of messages praising B that the consumer receives. Since the consumer engages in sampling, the number of each particular message that he receives is stochastic. The firm controls the mean of the distribution of the number of messages received. Thus, assume that $\tilde{X}(\mu^{C>B})$ ~ exponential with mean = $\overline{X}_i^C$

{ $P([\tilde{X}(\mu^{C>B}) = x]) = \frac{1}{\overline{X}_i^C}\exp(-\frac{x}{\overline{X}_i^C})$ } where *i* is the State; $\tilde{X}(\mu^{B>C})$ ~ exponential with

mean = $\overline{X}_i^B$. As before, the mean of the distribution of the number of messages received is a sum of the firm's effort (λ_i^C, λ_i^B) and the effort exerted by the unbiased recommenders (λ^U): { $\overline{X}_1^C = \lambda_1^C + \lambda^U$, $\overline{X}_2^C = \mu_2^C = \lambda_2^C, \overline{X}_1^B = \lambda_1^B$, $\overline{X}_2^B = \lambda_2^B + \lambda^U$ }. Similarly to the previous specification, the costs that the firm bears is quadratic in its effort.

Upon receiving { $\tilde{X}(\mu^{C>B}), \tilde{X}(\mu^{B>C})$ }, the consumer will buy C iff

$\frac{P(\tilde{X}(\mu^{C>B}), \tilde{X}(\mu^{B>C}) \,|\, state = 1, \overline{X}_1^C, \overline{X}_1^B)}{P(\tilde{X}(\mu^{C>B}), \tilde{X}(\mu^{B>C}) \,|\, state = 2, \overline{X}_2^C, \overline{X}_2^B)} > \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)}$. To simplify the calculations

below, assume that $\frac{P(s2)V^B}{P(s1)(V_H^C - V^B)} = 1$. Thus, the consumer will buy C iff

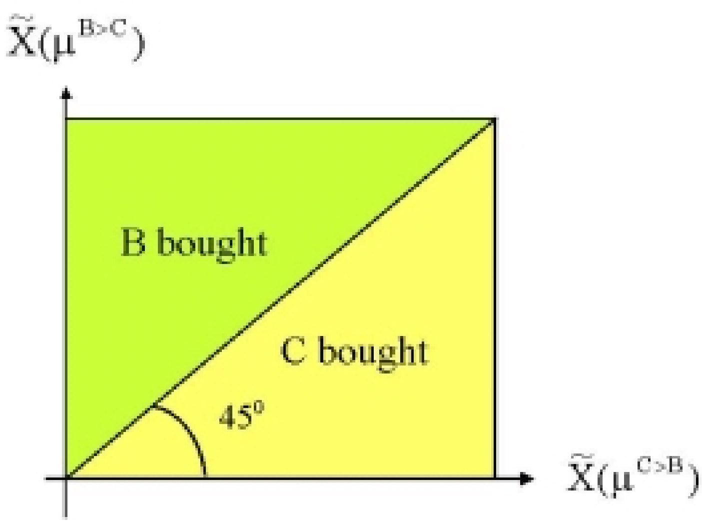
$P(\tilde{X}(\mu^{C>B}), \tilde{X}(\mu^{B>C}) \,|\, state = 1, \overline{X}_1^C, \overline{X}_1^B) > P(\tilde{X}(\mu^{C>B}), \tilde{X}(\mu^{B>C}) \,|\, state = 2, \overline{X}_2^C, \overline{X}_2^B)$.

Substituting for the expression of the exponential p.d.f.,

$\frac{\overline{X}_2^B \overline{X}_2^C}{\overline{X}_1^C \overline{X}_1^B}\exp(-\frac{\tilde{X}(\mu^{C>B})}{\overline{X}_1^C} - \frac{\tilde{X}(\mu^{B>C})}{\overline{X}_1^B}) > \exp(-\frac{\tilde{X}(\mu^{C>B})}{\overline{X}_2^C} - \frac{\tilde{X}(\mu^{B>C})}{\overline{X}_2^B})$. Let us focus on the symmetric equilibrium where $\overline{X}_1^C = \overline{X}_2^B$, $\overline{X}_2^C = \overline{X}_1^B$. This implies that the consumer chooses C iff $\tilde{X}(\mu^{C>B})(\frac{1}{\overline{X}_2^C}-\frac{1}{\overline{X}_1^C}) > \tilde{X}(\mu^{B>C})(\frac{1}{\overline{X}_1^B}-\frac{1}{\overline{X}_2^B})$. It is easy to see that the only possible equilibrium is one where $\{\overline{X}_1^C > \overline{X}_2^C, \overline{X}_1^B < \overline{X}_2^B\}$.

To show that no other possible efforts can be an equilibrium, consider an equilibrium where $\{\overline{X}_1^C < \overline{X}_2^C, \overline{X}_1^B < \overline{X}_2^B\}$ This would imply that the consumer buys C whenever $\tilde{X}(\mu^{C>B}) + \tilde{X}(\mu^{B>C}) < 0$. This of course implies that the consumer always buys B. Thus, C finds it optimal not to expend any effort on promotion, which results in a contradiction: $\overline{X}_1^C = \lambda^U > \overline{X}_2^C$. The figure below demonstrates the region where the consumer chooses C:

Figure 9: Consumer's Decision Based on Number of Messages Received



If we integrate the region above, we see that Pr(C is bought) = $\frac{\overline{X}_i^C}{\overline{X}_i^C + \overline{X}_i^B}$. This is exactly equivalent to the symmetric specification we had discussed at the beginning of the section. Thus, we see that a different model that allows multiple messages to be received can yield the same results as the simple symmetric model we had posed at the beginning.

5b. Price Signaling

In the main model we assume that prices contain no information concerning the quality of the products (the state of the world). Thus, in our model the consumers can only infer quality from the chat and not from the prices. However, we have often seen in both the marketing as well as the economic literature that prices can serve as signals of quality. See, for example, Kalra, Rajiv, and Srinivasan (1998) and Anderson & Simester (2000). In our model, can prices possibly signal to consumers which products they should buy?

We next extend our main model to include possible price signaling, a separating equilibrium where firms post different prices depending on the state of the world. Thus, let us suppose that firms fix prices at the beginning of the game. A consumer may receive a message, as before, and draw an inference on the state of the world. With a positive probability, the consumer remains unaware of the entrant if he does not receive any messages. An aware consumer observes the incumbent's as well as the entrant's prices: $[P_1^C, P_1^B]$ in State 1 and $[P_2^C, P_2^B]$ in State 2 (the superscript refers to the player, and the subscript refers to the state of the world). The prices are informative only in case where $[P_1^C, P_1^B] \neq [P_2^C, P_2^B]$. Note that we do not require all elements of the vectors to differ. Thus, we are allowing the case where $P_1^C \neq P_2^C$ but $P_1^B = P_2^B$. If we have separation in prices, the aware consumer becomes perfectly informed about the state of the world after viewing the prices. A consumer who is unaware of the entrant only observes the price of the incumbent: P_1^B in State 1, and P_2^B in State 2. Note that if in equilibrium $P_1^B \neq P_2^B$, the unaware consumer can infer that an entrant exists and, moreover, becomes perfectly informed about the quality of the entrant.

We show that no price signaling equilibrium can exist in our model. The reason is that prices in our model are not credible signals of quality. Thus, in the price signaling literature, the high quality type is able to separate herself from the ghost low quality type by charging a higher price. Since the low quality type bears higher losses in increasing the price, high price can serve as a credible signal of quality. Our model, on the other hand, lacks any of the elements that can facilitate price signaling. Thus, in a one-period model with homogeneity of preferences and zero marginal cost, both types can only benefit from a hike in price. Thus, no separation exists. The only additional difficulty in our model as opposed to most of the is that signaling is usually done in the context of a monopolist firm. Here, however, we must consider possible price signaling in a duopoly.

Lemma 1
There does not exist a price signaling equilibrium in the model above. Thus, there does not exist a separating equilibrium where the consumer observes a pair of prices $[P_1^C, P_1^B]$ in State 1 and a pair of prices $[P_2^C, P_2^B]$ in State 2, where $[P_1^C, P_1^B] \neq [P_2^C, P_2^B]$.

See Appendix for details of the proof. The example below illustrates the idea behind the proof. Consider a price-separating equilibrium where the consumer sees prices $[P_1^C, P_1^B] = [V_H^C - V^B, 0]$ in State 1, and $[P_2^C, P_2^B] = [0, V^B]$ in State 2. Note that these are the duopoly prices that would prevail under perfect information. Thus, we see that the aware consumer buys C in State 1, and B in State 2. The unaware consumer only observes B's prices, but can nonetheless infer the state of the world and makes the same purchase decisions as the aware consumer since $P_1^B \neq P_2^B$.

Let us examine off-path beliefs that can support the equilibrium above. Note that to keep C from deviating in State 2, the consumer must infer that the state of the world is **2** after observing prices $[V_H^C - V^B, V^B]$. However, to keep B from deviating in State 1, the consumer must infer that the state of the world is **1** following observing prices $[V_H^C - V^B, V^B]$. Since it is impossible to maintain beliefs that satisfy both of these conditions, we arrive at a contradiction. Thus, a separating equilibrium in prices does not exist in our model.

5c. Mixing Equilibrium

Let us focus more intently on the result that the firms lie in equilibrium or spend more resources promoting an inferior product. Let us suppose that we are in a situation where the ex-ante probability that the entrant is superior is low, P(s1) is low, as is N^U. Thus, there are few unbiased reviews and the entrant is most certainly of inferior quality. Would that imply that in expectation C would invest lots of resources into disseminating false information?

This does not turn out to be the case. Note that when P(s1) is very low, it will be "hard" to convince a consumer to buy C. More formally, $\frac{P(\mu^{C>B} \mid \text{state} = 1)}{P(\mu^{C>B} \mid \text{state} = 2)} < \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)}$.

Since no pure strategy equilibrium exists, we can consider a mixing equilibrium. Specifically, let us consider an equilibrium where the consumer mixes between buying C and B upon receiving $\mu^{C>B}$, and buys B otherwise. Thus, the consumer buys C with probability δ upon receiving $\mu^{C>B}$. In our model, this is equivalent to multiplying the profit parameter by the mixing equilibrium = $\rho\delta$. In this equilibrium,

$\frac{P(\mu^{C>B} \mid state = 1)}{P(\mu^{C>B} \mid state = 2)} = \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)}$. We can show that the expression

$\frac{P(\mu^{C>B} \mid state = 1)}{P(\mu^{C>B} \mid state = 2)}$ is decreasing in the profit parameter. As we decrease P(s1), the expression on the right increases. This implies that the profit parameter must decrease, or δ must decrease. This in turn implies a dampening of promotional activity by both B and C. This agrees with our intuition that we should not get inundated with false messages by the firm.

There are a number of extensions that can be pursued in the future:

1) We can further think about endogenizing some other parameters. One natural extension would be to endogenize the prior, P(s1), parameter by postulating that there is a cost of entry. If we consider an extension where there are many types of quality entrants, we will get the result that the very low quality types will not enter since they will make very little profits.
2) Another natural candidate to be treated as an endogenous variable is the number of unbiased reviews. Thus, we might think that in a multi-period model firms can

influence the size of N^U by free trial. (Consider a recent news item where a movie studio refused to screen a movie for critics ostensibly not to reveal important plot points.)

3) In our model there is homogeneity of preferences. We can show that preference heterogeneity does not change the results as long as the niche segment is not too large.

4) We can look at another game form: a game form with advertising but no anonymity. Note that this is the setting that is equivalent to offline advertising. This is a signaling model where consumers interpret advertising as a signal of quality. Note that in our set-up, a firm has more incentive to advertise its inferior product. Thus, $\mu^{C>B}$ message is actually an indicator of poor quality of C. However, if P(s1) is high enough, and as long as the signal is sufficiently imprecise, the consumers may still choose to buy C following $\mu^{C>B}$. Thus, this format would motivate C to advertise simply to increase product awareness in the context where consumers are positively pre-disposed to entrants. Note that the logic here is different from a classic signaling model since here advertising is an imprecise negative signal, but the firm still chooses to advertise to increase awareness.

Let us conclude with the following thought. What is the fundamental question that this paper addresses? We explore a new advertising context: a setting where advertising and word of mouth become perfect substitutes since to the consumer they appear indistinguishable. We find that in this context consumers still benefit from chat, despite the fact that firms choose to lie. Moreover, the consumers may benefit from such a system compared to a regulated system.

References

1) Anderson, E.; Simester, D. (2000) "Price Discrimination as an Adverse Signal: Why an Offer to Spread Payments May Hurt Demand," MIT working paper.

2) Avery, C.; Resnick, P.; Zeckhauser, R. (1999) "The Market for Evaluations," *American Economic Review*. 89 (June), 564-84.

3) Banerjee, A. (1992) "A Simple Model of Herd Behavior," *The Quarterly Journal of Economics*. 107 (3), 797-817.

4) Banerjee, A. (1993) "The Economics of Rumours," *The Review of Economic Studies*. 60 (2), 309-27.

5) Dixit, A.; Norman, V. (1978) "Advertising and Welfare," *The Bell Journal of Economics.* 9 (1), 1-17.

6) Dodson, J.; Muller, E. (1978) "Models of New Product Diffusion Through Advertising and Word-of-Mouth," *Management Science.* 24 (15), 1568-1578.

7) Ellison, G.; Fudenberg, D. (1995) "Word-of-Mouth Communication and Social Learning," *The Quarterly Journal of Economics*. 110 (1), 93-125.

8) Feinberg, F. (1992) "Pulsing Policies for Aggregate Advertising Models," *Marketing Science*. 11 (3), 221-234.

9) Horstmann, I.; Moorthy, S. (2000) "Advertising and Quality for Services," *Working paper*, University of Toronto.

10) Kihlstrom, R.; Riordan, M. (1984) "Advertising as a Signal," *Journal of Political Economy.* 92 (3), 427-450.

11) Kalra, A.; Rajiv, S., Srinavasan, K. (1998) "Response to Competitive Entry: A Rationale for Delayed Defensive Reaction," *Marketing Science*. 17 (4), 380-405.

12) Little, J. (1979) "Aggregate Advertising Models: The State of the Art," *Operations Research.* 27 (4), 629-667.

13) Little, J.; Lodish, L. (1969) "A Media Planning Calculus," *Operations Research.* 17 (1), 1-35.

14) Mahajan, V.; Muller, E., Kerin, R. (1984) "Introduction Strategy for New Products With Positive and Negative Word-of-Mouth," *Management Science*. 30 (12), 1389-1404.

15) Mahajan, V.; Muller, E. (1986) “Advertising Pulsing Policies For Generating Awareness for New Products,” *Marketing Science*. 5 (2), 89-106.

16) McFadden, D.; Train, K. (1996) “Consumers' Evaluation of New Products: Learning from Self and Others,” *Journal of Political Economy.* 104 (4), 683-703.

17) Monahan, G. (1984) “A Pure Birth Model of Optimal Advertising With Word-of-Mouth,” *Marketing Science.* 1 (2), 169-178.

18) Nelson, P. (1974) “Advertising as Information,” *Journal of Political Economy*. 81 (July/August), 729-754.

19) Simon, H. (1982) “ADPULS: An Advertising Model with Wearout and Pulsation,” *Journal of Marketing Research*. 19 (August), 352-63.

20) Sasieni, M. (1971) “Optimal Advertising Expenditure,” *Management Science*. 18 (4), 64-72.

21) Stegeman, M. (1991) “Advertising in Competitive Markets,” *The Rand Journal of Economics.* 81 (1), 210-223.

22) Vettas, N. (1997) “On the Informational Role of Quantities: Durable Goods and Consumers' Word-of-Mouth Communication,” *International Economic Review*. 38 (4), 915-944.

23) Vidale, M.; Wolfe, H. (1957) “An Operations-Research Study of Sales Response To Advertising,” *Operations Research*. 5, 370-381.

Appendix

Proposition 1

Since the consumer's decision is binary (whether or not to buy the entrant based on a message), let us start with the four possible equilibrium strategies by the consumer. Note that the consumer buys B following μ^{B} by assumption.

1) Suppose that the consumer buys the entrant's product following both $\mu^{B>C}$ and $\mu^{C>B}$. This implies that the incumbent only makes profit when μ^{B} message is drawn. But this would imply that the incumbent would not have any incentives to message, which in turn implies that $\mu^{B>C}$ message is perfectly informative. (This is due to the fact that $\mu^{B>C}$ message has to come from the unbiased source). Since $\mu^{B>C}$ message is perfectly informative, the consumer should buy B following $\mu^{B>C}$ which contradicts our initial assumption. This set of beliefs cannot be consistent.
2) Let us next suppose that the consumer buys the incumbent's product following both $\mu^{B>C}$ and $\mu^{C>B}$. Similarly, this cannot be in equilibrium since the entrant would not message, and $\mu^{C>B}$ becomes perfectly informative.
3) Consider an equilibrium where the consumer buys C following $\mu^{B>C}$ and buys B following $\mu^{C>B}$. Here we get a corner solution since both the B & C would not want to message. (Note that here the assumption that firm B can only send $\mu^{B>C}$ message & firm C can only send $\mu^{C>B}$ message is important). However, since neither biased sources are messaging, the signals becomes perfectly informative which would imply that the consumer should buy B following $\mu^{B>C}$ and should buy C following $\mu^{C>B}$. This is a contradiction of our earlier assumption.
4) Next, consider an equilibrium where the consumer buys B following $\mu^{B>C}$ and buys C following $\mu^{C>B}$. This results in the following four maximization problems for the firms (where G_i^C is C's net profit function in State i and G_i^B is B's net profit function in State i):

Entrant (C) in State 1: $$\max_{N_1^C} G_1^C(N_1^C, N_1^B) \equiv \max_{N_1^C} \frac{N_1^C + N^U}{N_1^C + N^U + N_1^B + N^0}\rho - \frac{(N_1^C)^2}{2} \qquad \textbf{(1)}$$
$$\text{s.t.} N_1^C \geq 0$$

Incumbent (B) in State 1: $$\max_{N_1^B} G_1^B(N_1^C, N_1^B) \equiv \max_{N_1^B} \frac{N_1^B + N^0}{N_1^C + N^U + N_1^B + N^0}\rho - \frac{(N_1^B)^2}{2} \qquad \textbf{(2)}$$
$$\text{s.t.} N_1^B \geq 0$$

Entrant (C) in State 2: $$\max_{N_2^C} G_2^C(N_2^C, N_2^B) \equiv \max_{N_2^C} \frac{N_2^C}{N_2^C + N^U + N_2^B + N^0}\rho - \frac{(N_2^C)^2}{2} \qquad \textbf{(3)}$$
$$\text{s.t.} N_2^C \geq 0$$

Incumbent (B) in state 2: $$\max_{N_2^B} G_2^B(N_2^C, N_2^B) \equiv \max_{N_2^B} \frac{N^U + N_2^B + N^0}{N_2^C + N^U + N_2^B + N^0}\rho - \frac{(N_2^B)^2}{2} \qquad \textbf{(4)}$$
$$\text{s.t.} N_2^B \geq 0$$

Note that we constrain the firms' actions to be positive. This could introduce the complications of corner solutions. However, it turns out that we can show that if $N_1^B \geq 0$, C would choose $N_1^C \geq 0$ and vice versa. (We can show this by looking at the firms' reaction functions). Thus, we do not need to worry about corner solutions in the relevant region. In addition, we can show that

$$\frac{\partial^2 G_1^C}{\partial^2 N_1^C} = -\frac{2N_1^B}{(N_1^C + N_1^B + N^U + N^0)^3}\rho - 1 < 0 \ ; \frac{\partial^2 G_1^B}{\partial^2 N_1^B} = -\frac{2(N_1^C + N^U)}{(N_1^C + N_1^B + N^U + N^0)^3}\rho - 1 < 0$$

$$\frac{\partial^2 G_2^C}{\partial^2 N_2^C} = -\frac{2(N_2^B + N^U + N^0)}{(N_2^C + N_2^B + N^U + N^0)^3}\rho - 1 < 0 ; \frac{\partial^2 G_2^B}{\partial^2 N_2^B} = -\frac{2N_2^C}{(N_2^C + N_2^B + N^U + N^0)^4}\rho - 1 < 0$$

This insures a unique maximum in the relevant region. Hence, our strategy for obtaining the solution is to solve using FOCs and ignore the constraint. We later discard any negative solutions as infeasible.

The four resulting FOCs are listed below:

Entrant (C) in State 1: $(N_1^B + N^0)\rho = N_1^C (N_1^B + N_1^C + N^U + N^0)^2$ **(5)**

Incumbent (B) in State 1: $(N_1^C + N^U)\rho = N_1^B (N_1^B + N_1^C + N^U + N^0)^2$ **(6)**

Entrant (C) in State 2: $(N_2^B + N^U + N^0)\rho = N_2^C (N_2^B + N_2^C + N^U + N^0)^2$ **(7)**

Incumbent (B) in State 2: $N_2^C \rho = N_2^B (N_2^B + N_2^C + N^U + N^0)^2$ **(8)**

We next show that there exists a unique positive solution to the equations (5) & (6). If we add (5) & (6) and simplify, we get the expression $\rho = (N_1^B + N_1^C)(N_1^B + N_1^C + N^U + N^0)$ **(9)**

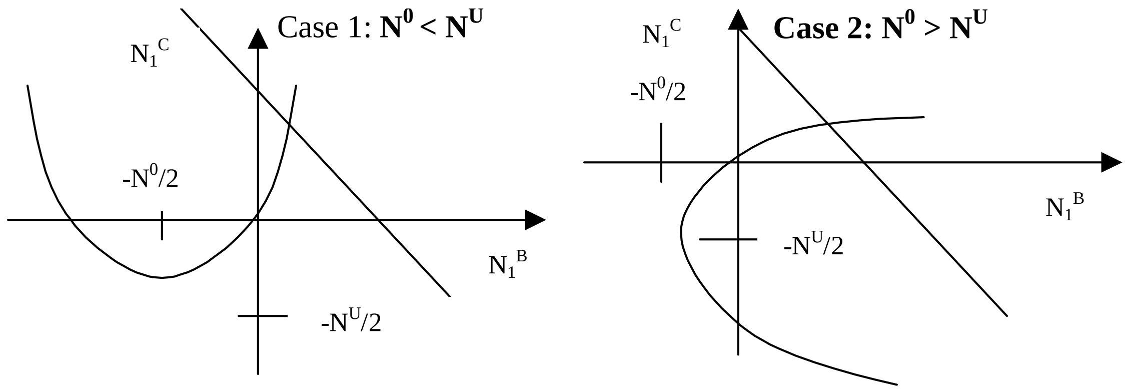
From this we can solve to get, $N_1^B + N_1^C = \dfrac{-(N^U + N^0) + \sqrt{(N^U + N^0)^2 + 4\rho}}{2} = w > 0$ **(10)**

This is the only positive solution. On the other hand, if we divide (5) by (6) (we can do this since we can see that $N_1^B = N_1^C = 0$ is not a solution to the equation), we get

$N_1^B (N_1^B + N^0) = N_1^C (N_1^C + N^U)$ **(11)**

Note that any solution that satisfies (5) & (6) must also satisfy (10) & (11). Thus, we are not losing any solutions. Graphically, we see that (10) describes a line on the (N_1^B, N_1^C) plane, whereas (11) defines a hyperbola. (To see this more clearly, we can re-write (11) as $\dfrac{(N_1^B - (-N^0/2))^2}{\frac{(N^0)^2 - (N^U)^2}{4}} - \dfrac{(N_1^C - (-N^U/2))^2}{\frac{(N^0)^2 - (N^U)^2}{4}} = 1$) The intersection of (10) & (11) give us the equilibrium values. The hyperbola can have two orientations depending on the relative size of N^U and N^0 parameters. From the graphs below, we see that there exists a unique positive solution:

Figure 1A: Existence and uniqueness of the solution



This is guaranteed since the hyperbola goes through the origin & the line $N_1^B + N_1^C = w > 0$ has a positive intercept term. Similarly, we can show uniqueness and existence for (N_2^B, N_2^C).

The explicit solutions are listed below.

$$N_1^B = \frac{1}{2}\left[\frac{2\rho + (N^0)^2 + N^U N^0 - N^0\sqrt{(N^U+N^0)^2+4\rho}}{\sqrt{(N^U+N^0)^2+4\rho}}\right]; \tag{12}$$

$$N_1^C = \frac{1}{2}\left[\frac{2\rho + (N^U)^2 + N^U N^0 - N^U\sqrt{(N^U+N^0)^2+4\rho}}{\sqrt{(N^U+N^0)^2+4\rho}}\right]; \tag{13}$$

$$N_2^B = \frac{1}{2}\left[\frac{2\rho + (N^U+N^0)^2 - (N^0+N^U)\sqrt{(N^U+N^0)^2+4\rho}}{\sqrt{(N^U+N^0)^2+4\rho}}\right]; \tag{14}$$

$$N_2^C = \left[\frac{\rho}{\sqrt{(N^U+N^0)^2+4\rho}}\right] \tag{15}$$

All the expressions above are greater than 0. Note the following two results:

1) $N_2^C - N_1^C = \frac{1}{2}N^U\left[1 - \frac{N^U+N^0}{\sqrt{(N^U+N^0)^2+4\rho}}\right] = \Delta$ Note that $0 < \Delta < \frac{N^U}{2}$. Thus, the entrant (C) spends more on promotion when his product is inferior, but $N_2^C < N_1^C + N^U$. That is, the consumer is more likely to hear $\mu^{C>B}$ when the entrant is superior.

2) $N_1^B - N_2^B = \frac{1}{2}N^U[1-\frac{N^U+N^0}{\sqrt{(N^U+N^0)^2+4\rho}}] = \Delta$. Similarly, the incumbent (B) spends more on promotion when his product is inferior, but $N_1^B < N_2^B + N^U$. That is, the consumer is more likely to hear $\mu^{B>C}$ when the incumbent is superior. From this we clearly see that the firm makes more profit when its product is superior since then it spends less money on promotion and is more likely to sell.

Next, let us turn back to consumer's problem and check that his decision rule is optimal, given the firms' actions. As we previously discussed, the consumer will optimally follow the recommendation system iff

$$\frac{P(\mu^{C>B}\,|\,s1)}{P(\mu^{C>B}\,|\,s2)} \geq \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)} \;\;\&\;\; \frac{P(\mu^{B>C}\,|\,s1)}{P(\mu^{B>C}\,|\,s2)} < \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)} \qquad \textbf{(7)}$$

(Note that if the inequalities in **(7)** do not hold, the firm's action will not affect the consumer's decisions. This contradicts our initial assumption and is not an equilibrium). Let us substitute the conditional probabilities that we obtain above. Here we see that

$$\frac{P(\mu^{C>B}\,|\,s1)}{P(\mu^{C>B}\,|\,s2)} = \frac{N_1^C + N^U}{N_2^C} \qquad \textbf{(16)}$$

$$\frac{P(\mu^{B>C}\,|\,s1)}{P(\mu^{B>C}\,|\,s2)} = \frac{N_1^B}{N_2^B + N^U} \qquad \textbf{(17)}$$

(We use the result that $N_1^B + N_1^C = N_2^B + N_2^C$ to simplify the fraction above). Note that since we showed that $N_1^B < N_2^B + N^U$ & $N_2^C < N_1^C + N^U$ → **(16)** > **(17).** We can also show that

$\frac{\partial[\frac{N_1^C + N^U}{N_2^C}]}{\partial\rho} < 0$, while $\frac{\partial[\frac{N_1^B}{(N_2^B + N^U)}]}{\partial\rho} > 0$. Thus, as long as 0 < P(S1) < 1 & $V_H^C > V^B$, we can find $\hat{\rho}$ s.t. $\forall\rho \leq \hat{\rho}$, **(7)** is guaranteed to hold. Thus, we showed the existence of a unique equilibrium. QED.

Proposition 2: Linear Costs

Steps 1-3 of the proof of the proposition do not depend on the convexity of costs and can be repeated here. Let us here explore the remaining possible equilibrium where the consumer buys C following [C>B] and buys B following [B>C]. The firms' maximizations are:

Entrant (C) in state 1: $$\max_{N_1^C} \frac{N_1^C + N^U}{N_1^C + N^U + N_1^B + N^0}\rho - N_1^C \quad \textbf{(18)}$$

Incumbent (B) in state 1: $$\max_{N_1^B} \frac{N_1^B + N^0}{N_1^C + N^U + N_1^B + N^0}\rho - N_1^B \quad \textbf{(19)}$$

Entrant (C) in state 2: $$\max_{N_2^C} \frac{N_2^C}{N_2^C + N^U + N_2^B + N^0}\rho - N_2^C \quad \textbf{(20)}$$

Incumbent (B) in state 2: $$\max_{N_2^B} \frac{N^U + N_2^B + N^0}{N_2^C + N^U + N_2^B + N^0}\rho - N_2^B \quad \textbf{(21)}$$

Of course, once again we impose the constraint that none of the firms' actions are negative. If we ignore the constraint, then we can obtain the firms' reaction functions from the maximizations above. The reaction functions are the following:

$$N_1^C = \sqrt{(N_1^B + N^0)\rho} - N_1^B - N^U - N^0 \quad \textbf{(22)}$$

$$N_1^B = \sqrt{(N_1^C + N^0)\rho} - N_1^C - N^U - N^0 \quad \textbf{(23)}$$

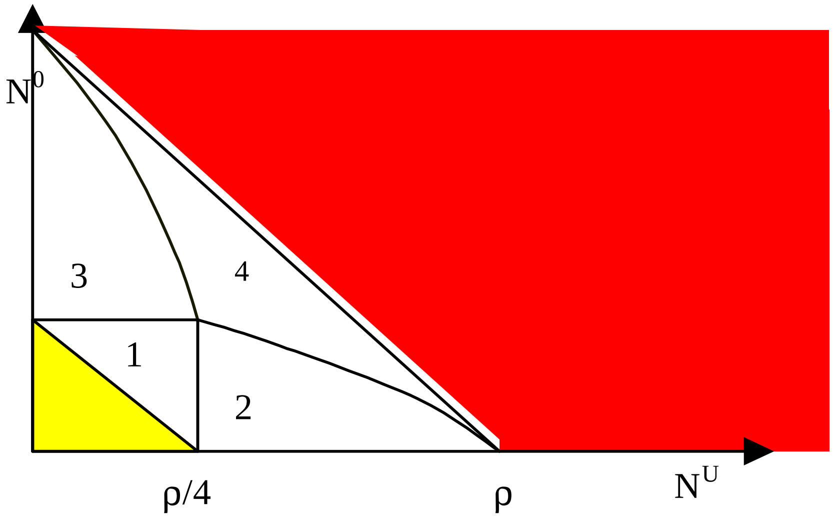
$$N_2^C = \sqrt{(N_2^B + N^U + N^0)\rho} - N_2^B - N^U - N^0 \quad \textbf{(24)}$$

$$N_2^B = \sqrt{N_2^C\rho} - N_2^C - N^U - N^0 \quad \textbf{(25)}$$

The above equations have one (possibly) positive solution:

$\{ N_1^C = \frac{\rho}{4} - N^U; N_1^B = \frac{\rho}{4} - N^0; N_2^C = \frac{\rho}{4}; N_2^B = \frac{\rho}{4} - N^0 - N^U \}$. We can see that this is a positive solution when $N_2^B = \frac{\rho}{4} - N^0 - N^U > 0$. Note that this is the case when ρ is relatively large. This is analogous to stating that the costs are relatively low. We can see that in this region there is no informative equilibrium since $N_1^C + N^U = N_2^C; N_1^B = N_2^B + N^U$. Thus, the content of the message has no informational value since the message is equally likely to have arrived from either of the two states of the world. In the language used earlier, we consider this a low cost region. In the Figure below, this region is the triangle in the lower left corner of the graph**.**

Figure 2A: Linear Costs



Next, let us consider the high cost region. In the region where $N^U + N^0 > \rho$, all firms choose to send 0 messages in both states of the world. (This is the region on the right side of the Figure).

In the intermediate cost region, $\frac{\rho}{4} < N^U + N^0 < \rho$, we have the following 4 regions:

Region 1: $\frac{\rho}{4} > N^U; \frac{\rho}{4} > N^0$

$$N_2^C = \sqrt{(N^U + N^0)\rho} - (N^U + N^0) < \frac{\rho}{4};\ N_2^B = 0;\ N_1^C = \frac{\rho}{4} - N^U;\ N_1^B = \frac{\rho}{4} - N^0$$

We can show that $N_2^C > N_1^C$ ($\frac{\rho}{4} < N^U + N^0$ and $\frac{\rho}{4} > N^0 \rightarrow N_2^C > \frac{\rho}{4} - N^U = N_1^C$) Thus, we can see that here the incumbent only spends on promotion when her product is of inferior quality, as does the entrant. Moreover, we can see that $N_2^C < N_1^C + N^U$ and

$N_1^B < N_1^B + N^U$ (since $\frac{\rho}{4} < N^U + N^0$) Thus, the messages are potentially informative.

Region 2: $\frac{\rho}{4} < N^U; \frac{\rho}{4} > N^0; \sqrt{N^U\rho} - (N^U + N^0) > 0$

$$N_2^C = \sqrt{(N^U + N^0)\rho} - (N^U + N^0) < \frac{\rho}{4};\ N_2^B = 0;\ N_1^C = 0;\ N_1^B = \sqrt{N^U\rho} - (N^U + N^0)$$

Here again we see that the firms lie. We can also see that there is potential for informativeness since $N_2^C < \frac{\rho}{4} < N^U = N_1^C + N^U$. We also see that

$N_1^B < N_2^C < N^U = N_2^B + N^U$

Region 3: $\frac{\rho}{4} > N^U; \frac{\rho}{4} < N^0; \sqrt{N^0\rho} - (N^U + N^0) > 0$

$N_2^C = \sqrt{(N^U + N^0)\rho} - (N^U + N^0) < \frac{\rho}{4}$; $N_2^B = 0$; $N_1^C = \sqrt{N^0\rho} - (N^U + N^0)$; $N_1^B = 0$

Here again we see that the entrant lies. We can easily see that here $N_1^B < N_2^B + N^U$. We can also show that here $N_2^C < N_1^C + N^U$. We need to show that

$\sqrt{(N^U + N^0)\rho} - \sqrt{N^0\rho} < U \rightarrow f = \sqrt{(N^U + N^0)\rho} - \sqrt{N^0\rho} - N^U < 0$. Note that

$\frac{\partial f}{\partial N^U} = \frac{\sqrt{\rho}}{2\sqrt{(N^U + N^0)}} - 1 < 0$ since $N^U + N^0 > \frac{\rho}{4}$. Also note that

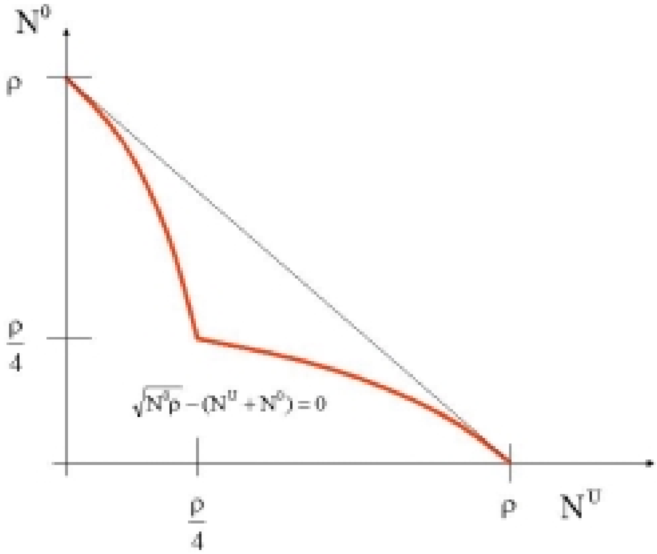
$\frac{\partial f}{\partial N^0} = \frac{\sqrt{\rho}}{2\sqrt{(N^U + N^0)}} - \frac{\sqrt{\rho}}{2\sqrt{N^0}} < 0$. Thus, f decreases as N^0 and N^U increase. Let's pick the smallest N^U and N^0 in the region: $N^0 = \frac{\rho}{4}, N^U = 0$. At this point, f=0. This is the maximum of f. Thus, we can see that in the relevant region, f < 0. This demonstrates informativeness.

Region 4: $\sqrt{N^0\rho} - (N^U + N^0) < 0; \sqrt{N^U\rho} - (N^U + N^0) < 0$

$N_2^C = \sqrt{(N^U + N^0)\rho} - (N^U + N^0)$; $N_2^B = N_1^B = N_1^C = 0$

Here again we see that the entrant lies. We can easily see that here $N_1^B < N_2^B + N^U$. We can also show that $N_2^C < N_1^C + N^U = N^U$. Similarly to the proof above, we need to show that $g = \sqrt{(N^U + N^0)\rho} - 2N^U - N^0 < 0$. The only difficulty here lies in the irregularity of region 4. It is graphed below in more detail:

Figure 3A: Region 4



Our strategy is to find the maximum value of *g* at the region above and show that it is 0.

We can show that $\frac{\partial g}{\partial N^0}=\frac{\sqrt{\rho}}{2\sqrt{(N^U+N^0)}}-1<0$ since $N^U+N^0>\frac{\rho}{4}$. Thus, for any point in the region 4, there exists a point on the border between A and C or A and B s.t. the point on the border results in a higher value of *g*. We thus can look for the maximum point on that border.

Let us next find the maximum value of *g* on that border curve. Let us first focus on the border between A and C. The equation for that curve is $\sqrt{N^0\rho}-(N^U+N^0)=0$. Let us solve for N^U in terms of N^0: $N^U=\sqrt{N^0\rho}-N^0$ and plug this into g and take a derivative w.r.t. N^0:

$$\frac{\partial g/\text{border}}{\partial N^0}=\sqrt[3/4]{\frac{\rho}{N^0}}-2\sqrt{\frac{\rho}{N^0}}+4$$

Since $\frac{\rho}{4}<N^0<\rho$, we see that the expression above > 0. Thus, the *g* function is increasing in N^0 on the border. The maximum point is, therefore at $N^0=\rho, N^U=0$. Of course, at this point g = 0.

Next, let us consider the curve between A and B. Once again, the equation for that curve is $\sqrt{N^U \rho} - (N^U + N^0) = 0$. We can solve for N in terms of N^U: $N^0 = \sqrt{N^U \rho} - N^U$.

$N^U = \sqrt{N^0 \rho} - N^0$. Let us plug this into g and take a derivative w.r.t. N^U:

$\frac{\partial g / \text{border}}{\partial N^U} = \sqrt[3/4]{\frac{\rho}{N^U}} - 2\sqrt{\frac{\rho}{N^U}} - 4 < 0$ since $\frac{\rho}{4} < N^U < \rho$. Thus, *g* is decreasing in N^U,

and the maximum of *g* is at $N^0 = \frac{\rho}{4}, N^U = \frac{\rho}{4}$; *g* at that point is < 0. QED.

Proposition 3

Let us compare the customer's expected value between the two game forms: the basic model (which we denote by *BM*) versus a game form where no advertising by either firm is allowed (which we denote by *NA*).

$$\text{Value}^{BM} = P(s1)(P^{BM}(\mu^{C>B} | s1)V_H^C + (1 - P^{BM}(\mu^{C>B} | s1))V^B) + P(s2)(1 - P^{BM}(\mu^{C>B} | s1))V^B - P$$ **(26)**

$$\text{Value}^{NA} = P(s1)(P^{NA}(\mu^{C>B} / s1)V_H^C + (1 - P^{NA}(\mu^{C>B} / s1))V^B) + P(s2)V^B - P \qquad \textbf{(27)}$$

Note that in the basic model, in equilibrium the consumer can hear signal $\mu^{C>B}$ in either state of the world. Of course, hearing this signal in state 2 will cause the consumer to buy C and derive no value from the purchase. On the other hand, in the system with no advertising, the consumer will never hear $\mu^{C>B}$ in state 2. On the other hand, the consumer may also be less likely to hear signal $\mu^{C>B}$ in state 1 in the system with no advertising. (Since C will not be promoting its product in state 1). Thus, it is the tradeoff between these two forces that determines which system is better.

$$\frac{P^{BM}(\mu^{C>B} / s1) - P^{NA}(\mu^{C>B} / s1)}{P^{BM}(\mu^{C>B} / s2)} \geq \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)} \qquad \textbf{(28)}$$

In order to simplify the expression above, we need to substitute for the expressions below:

$$P^{BM}(\mu^{C>B} | s1) = \frac{N_1^C + N^U}{N_1^C + N_1^B + N^U + N^0} \qquad \textbf{(29)}$$

$$P^{BM}(\mu^{C>B} \mid s2) = \frac{N_2^C}{N_2^C + N_2^B + N^U + N^0} \quad \textbf{(30)}$$

$$P^{NA}(\mu^{C>B} \mid s2) = \frac{N^U}{N^U + N^0} \quad \textbf{(31)}$$

After simplification, we obtain the following expression for the left-hand side of (28):

$$\frac{P^{BM}(\mu^{C>B} \mid s1) - P^{NA}(\mu^{C>B} \mid s1)}{P^{BM}(\mu^{C>B} \mid s2)} = \frac{N^0 N_1^C - N^U N_1^B}{(N^U + N^0)N_2^C} = \frac{N^0 - N^U}{N^U + N^0}$$

Thus, as long as $\frac{N^0 - N^U}{N^U + N^0} > \frac{P(s2)V^B}{P(s1)(V_H^C - V^B)}$, the consumer expects to be better off in the game where advertising is allowed. Note that $\frac{N^0 - N^U}{N^U + N^0}$ measures the signal-to-noise ratio in the model with no advertising.

Alternative Specifications

Symmetric Specification: "Seeding"

Consider the "seeding" specification where $T_1^C = \kappa N_1^C + N^U$, $T_2^C = N_2^C$, $T_1^B = N_1^B$, and $T_2^B = \kappa N_2^B + N^U$. Parts 1-3 of proof of Proposition 1 still hold. The only candidate for equilibrium is one where the consumer buys C following $\mu^{C>B}$ and B following $\mu^{B>C}$. Thus, we can derive the probabilities of getting either of the messages:

$$P(\mu^{C>B} \mid s1) \equiv f(T_1^C, T_1^B) = \frac{T_1^C}{T_1^C + T_1^B};\ P(\mu^{C>B} \mid s2) \equiv f(T_2^C, T_2^B) = \frac{T_2^C}{T_2^C + T_2^B}$$

$$P(\mu^{B>C} \mid s1) \equiv f(T_1^C, T_1^B) = \frac{T_1^B}{T_1^C + T_1^B};\ P(\mu^{B>C} \mid s2) \equiv f(T_2^C, T_2^B) = \frac{T_2^B}{T_2^C + T_2^B} \quad \textbf{(32)}$$

We can still show concavity of net profit functions: $\frac{\partial^2 G_1^C}{\partial^2 N_1^C} < 0$, etc. Thus, once again we turn our attention to the FOCs.

Due to the symmetry of the specification, we can see that $N_1^C = N_2^B$, $N_1^B = N_2^C$. Thus, we can consider the FOCs for State 1 only. We express the results in terms of T_1^C and T_1^B:

From C's maximization: $k\rho \frac{T_1^B}{(T_1^C + T_1^B)^2} = (\frac{T_1^C - N^U}{k})$ (33)

From B's maximization: $\rho \frac{T_1^C}{(T_1^C + T_1^B)^2} = T_1^B$ (34)

If we assume that $N^U > 0$, this implies that $T_1^C > 0$. From (34) we can see that this would imply that $T_1^B > 0$. Since $T_1^B > 0$, according to (33), $N_1^C = (\frac{T_1^C - N^U}{k}) > 0$.

If we divide (33) by (34) (since $T_1^B > 0$, we are dividing by a non-zero expression), we obtain the expression

$k^2 (T_1^B)^2 = T_1^C (T_1^C - N^U)$ (35)

We add $\frac{(N^U)^2}{2}$ to both sides of (35) to complete the square. After we substitute the expressions for T_1^B and T_1^C, we get

$(N_2^C)^2 = (N_1^C)^2 + (\frac{N^U}{k}) N_1^C$ (36)

From this, we can see that $N_2^C > N_1^C$ as long as $N^U > 0$. Also, $N_2^C - N_1^C$ decreases as k increases, and $N_2^C - N_1^C$ increases as N^U increases. Note also that $\lim_{k \to \infty} N_2^C - N_1^C = 0$.

(Same results hold for B by symmetry).

We can similarly demonstrate uniqueness of the solution.

Lemma 1

The following is a proof by contradiction. Thus, suppose that there exists a separating equilibrium in prices: aware consumer observes $[P_1^C, P_1^B]$ in state 1 and $[P_2^C, P_2^B]$ in state 2, where $[P_1^C, P_1^B] \neq [P_2^C, P_2^B]$. The unaware consumer observes $[P_1^B]$ in state 1 and $[P_2^B]$ in state 2. Let us assume that the unaware consumer can make an inference on the state of the world iff $P_2^B \neq P_1^B$.

Let us next consider the profits that the two firms make in the two states of the world: $\{\Pi_1^C, \Pi_1^B\}$ and $\{\Pi_2^C, \Pi_2^B\}$. Since C can deliver no value to the consumer in state 2, $\Pi_2^C = 0$. Let us apply the Pareto optimality criterion and only consider equilibria where B is bought in state 2. (Thus, we are ruling out equilbria where B charges such a high price in state 2 that the consumer buys neither B nor C. Such equilibria are Pareto dominated by an equilibrium where $P_2^B \leq V^B$ since both the consumer and B are better off, while C is indifferent). Thus, we have $\{\Pi_2^C, \Pi_2^B\} = \{0, P_2^B\}$. Similarly, let us rule out equilibria where neither good is bought in state 1 since these equilibria are Pareto dominated by those where at least one good is bought in state 1.

1) Let us suppose that an aware consumer buys C in state 1. This implies that $V^B - P_1^B < V_H^C - P_1^C$ since buying B always remains a choice for the consumer. Let's call the fraction of unaware consumers in state 1, α. Note that the consumer only buys B if $P_1^B = P_2^B = P^B$. Otherwise, he becomes aware of the entrant, learns the state of the world and chooses to buy C. (Note that under no beliefs would B be better off by charging $P_1^B > V^B$ in state 1 since even under most favorable beliefs this results in zero profit for B). Thus, if B pools on prices and charges $P_1^B = P_2^B$, the payoffs are $\{\Pi_1^C, \Pi_1^B\} = \{(1-\alpha)P_1^C, \alpha P^B\}$, $\{\Pi_2^C, \Pi_2^B\} = \{0, P^B\}$. If B separates on price and charges $P_1^B \neq P_2^B$, the payoffs are $\{\Pi_1^C, \Pi_1^B\} = \{P_1^C, 0\}$, $\{\Pi_2^C, \Pi_2^B\} = \{0, P_2^B\}$. We can see that in both cases, B is weakly better off in state 2, while C is weakly better off in state 1.

Suppose that B pools on prices: $P_1^B = P_2^B = P^B$. Note that there are no beliefs that support separation since C prefers to charge P_1^C in both states of the world and lead the consumer to infer that the true state of the world is state 1. Thus, we arrive at a contradiction.

Suppose that B does not pool on price: $P_1^B \neq P_2^B$. Note that since C weakly makes more profits in state 1, in order to support the equilibrium the consumer must believe that the **state of the world is 2** when encountering the off-path pair of prices $[P_1^C, P_2^B]$. Otherwise, C would charge P_1^C in state 2. To keep B from deviating and charging P_2^B in state 1, since B weakly makes more profit in state 2, the consumer must believe that **state 1** is in effect upon seeing $[P_1^C, P_2^B]$. Thus, we arrive at a contradiction.

2) Let us suppose that an aware consumer buys B in state 1. Thus C is never bought. This implies that B sets $P_1^B = P_2^B = V^B$. (Note that under no beliefs is B better off by charging $P_2^B > V^B$ in state 2 since even under most favorable beliefs this results in zero profit for B. Similarly, B can always improve his payoff by raising his price). The only way to obtain a separation here is by having C signal the state of the world. Since C makes 0 in both states of the world, let us assume that it prefers to pool on price. Thus, we see that no separating equilibrium can exist.