Adoption games with network effects: a generalized random graph model

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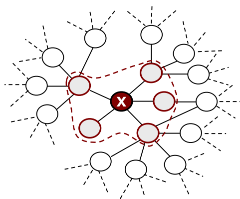
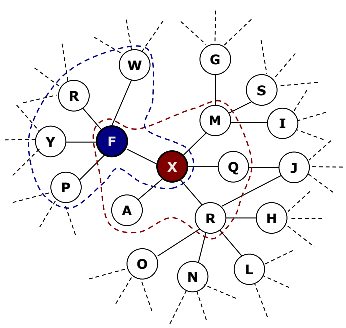


Overview

- Network effects are often "local"
  - Communication technologies, business networks, online marketplaces…
- The structure of underlying social or business networks affects the adoption of network goods
  - An agent's "local" network directly affects their value from adoption…
  - …but so does the structure of the rest of the social network
  - Local networks are connected
  - One's neighbors' local networks affect their adoption decisions

Overview: Illustrating local networks



Overview

- Agents in this kind of network generally have:
  - *good (perfect) information about the structure of their own local network*
  - *some information about the structure of the other local networks they belong to (their neighbor's local networks)*
  - *very little or no information about the exact structure of the rest of the social network*
- Many useful probabilistic abstractions of networks (graphs) have been developed recently
  - Newman, Watts, Strogatz (generalized random graphs)
  - Watts and Strogatz (small-world models)
  - Price, Albert and Barabasi (preferential attachment models)

Overview

- Objectives
  - To model costly adoption of a product with local network effects (a model of demand with local network effects)
  - To apply this model to study a bunch of research questions
- Progress thus far
  - Modeled adoption game with a single product, pretty general agent characteristics and network structure
  - Studied (briefly) what the structure of the "adoption networks" look like
  - Answered (or answering) some simple questions: monopoly, monopoly with free samples, monopoly with an installed base, duopoly with identical products, differentiated duopoly

Snapshot of some results

- Adoption game has at least one (and generally many) symmetric Bayesian Nash equilibria
  - All equilibria involve (generalized) threshold strategies
  - Equilibria can be strictly (Pareto) ordered, based on a simple parameter (equilibrium probability of neighbor adoption)
  - There is always a best equilibrium, which is "coalition proof"
  - Each Bayesian Nash equilibrium corresponds to a "fulfilled expectations" equilibrium, and vice versa
- Adoption networks have some interesting structural properties
- Some answers to other questions
  - Monopoly pricing is generally higher than a standard model that ignores network structure would predict
  - A monopolist always wants to give free versions to a fraction of their customers (if targeted, to low-degree customers)
  - The only duopoly equilibrium that is 'stable' involves marginal cost pricing

Model

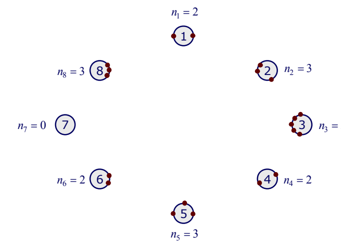
- Set of potential customers $K = \{1,2,3,\ldots,M\}$
- Single homogeneous network good that costs *c*
- Customers connected by an underlying social network (more on this in a couple of slides)
- Each customer has:
  - A neighbor set $N_k = \{N_{(k,1)}, N_{(k,1)}, \ldots, N_{(k,n_k)}\}$
  - A degree (number of neighbors) n_k
  - A type (index of valuation of product) $\theta_k \sim F$
- Each customer makes an adoption choice $a_k \in \{0,1\}$
- Value from adoption for customer *k:*

$$a_k[u(\sum_{i \in N_k} a_i, \theta_k) - c]$$

- More generally: any situation with local externalities

Model

- Social network: instance of generalized random graph with degree distribution $p(x), x \in \{0,1,2,\ldots,m\}$
- How are these graphs constructed?



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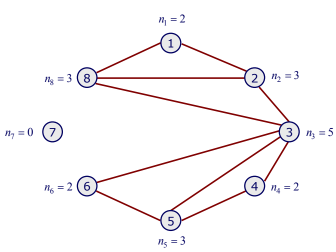


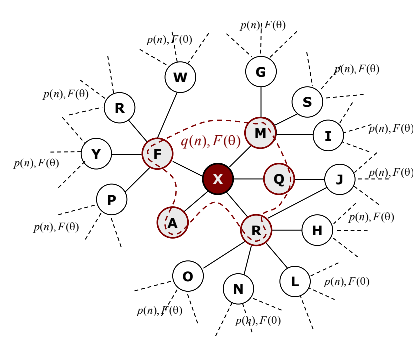
Model: Sequence of the game

- Nature creates the social network (according to the random graph algorithm), draws types for each agent
- Each agent *k* observes their type, their neighbor set, and (therefore) their degree
- Each agent *k* chooses either to adopt (a_k*=1*) or not (a_k*=0*)
- Payoffs are realized

Model: Information

- After each agent realizes their neighbor set and type:
  - They know the exact structure of their local network
  - They have very little information about the structure of the rest of the network
    - Posterior *p(x)* on degree of non-neighbors
  - They have inexact (but better) information about the structure of the local networks they belong to
    - Posterior *q(x)* on degree of neighbors
  - They know their type, do not know anyone else's type
    - Posterior *F* on all other agents
    - The results should hold for correlated degree, type

Model: Information



Model: Equilibria

- Each symmetric Bayesian Nash equilibrium involves a threshold strategy:

$$s(n_k, \theta_k) = \begin{cases} 0, & \theta_k < \theta^*(n_k) \\ 1, & \theta_k \geq \theta^*(n_k) \end{cases}$$

  with threshold $\theta^* = [\theta(1), \theta(2), \ldots, \theta(m)]$
- "No adoption" is always an equilibrium
- The equilibria can be ordered: $\Theta^* = \{\theta^A, \theta^B, \ldots\}$

$$\theta^A < \theta^B < \ldots$$

Model: Equilibria

- This ordering is based on the equilibrium probability of neighbor adoption

$$r(\theta^*) = \sum_{n=1}^{m} q(n)[1 - F(\theta^*(n))]$$

- "Higher" equilibria strictly Pareto-dominate lower ones, and therefore, there is a best equilibrium, which has the highest value of $r(\theta^*)$
  - Is coordination simpler if it is (a) local and (b) based on a simple parameter?
- Each "fulfilled expectations" outcome with expectation *r* has a corresponding Bayesian Nash equilibrium with $r(\theta^*) = r$
- The best equilibrium is the unique coalition-proof correlated equilibrium

Example: Complete social network

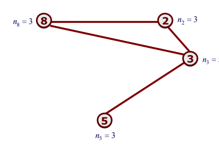
- *p(M–1)=1, p(n)=0 for n < (M—1)*
- Social network is complete graph
- This corresponds to a standard model
  - "Fulfilled expectations" equilibria with a continuum of types and customers always have an 'outcome equivalent' Bayesian Nash equilibrium in an *M*-player adoption game with heterogeneous types
  - Perhaps the latter is a better choice, because it allows one to examine stability more closely

Example: Pure random graph

- *F(1)=1, F(θ)=0* otherwise
- Adoption is completely determined by structure of the social network

$$s(n_k) = \begin{cases} 0, & n_k < n^* \\ 1, & n_k \geq n^* \end{cases}$$

- Structure of the "adoption network"



Structure of adoption networks

$\Phi_p(w) = \sum_{x=0}^{\infty} p(x) w^x$: moment-generating function of the degree distribution of the social network

$\Phi_\alpha(w) = \sum_{x=0}^{\infty} \alpha(x) w^x$: moment-generating function of the degree distribution of the **adoption** network

Then, for a pure random graph:

$$\Phi_\alpha(w) \cong \Phi_p(1 - \bar{Q}(\delta^*) + w\bar{Q}(\delta^*))$$

Summary