The effects of task and information asymmetry on business process redesign

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Abstract

The effective design of business processes is a subject of considerable importance to corporations today. Our research develops a theoretical framework for process design that is aimed at providing practical guidelines for process managers. The abundance of context-specific case studies which exist today share many success stories but provide little in terms of a general methodological approach. In this paper, we describe our general framework for the analysis and design of business processes. We outline a typical business process and critically evaluate typical pre- and post-reengineering process design issues. Explicit aspects of our analysis address workflow design, task bundling, technological enablers, and performance-based incentives. We examine the effects of task size asymmetry and performance information asymmetry on the optimal process design. Our results indicate that, with increased asymmetry, certain types of process designs become more desirable. Furthermore, we look at the interaction between job asymmetry and other process design factors such as knowledge intensity and level of job customization. Finally, we show when asymmetry can cause process reengineering efforts to complement the classic performance-based incentive compensation model. Practical implications of our results are illustrated for a variety of process design cases.

Keywords: Reengineering; Queuing; Organizational design; Incentives; Information systems design

1. Introduction

Business process reengineering has been around for over five years now. The changes advocated by the seminal books and papers in this area (Davenport and Short, 1990, Davenport, 1993, Hammer, 1990, Hammer and Champy, 1993) were supposed to increase productivity dramatically through a set of sweeping process modifications. Since these ideas were published, there have been a number of stories of their successful application; however, there have also been a number of reengineering failures. The failures are attributed partly to incorrect implementation of these ideas, and partly to the fundamental resistance to change that many organizations are believed to possess. The subsequent body of reengineering literature has been immense. A significant number of articles in the trade press, e.g. (Ballou, 1995, Brittain, 1994, Byrne, 1993, Champy, 1995) has presented case studies and outlined suggestions for implementation approaches. Academic studies have examined strategic (Short and Venkatraman, 1992), technological

(Earl et al., 1995; Stoddard and Jarvenpaa, 1995), organizational (Brickley et al., 1997; Whinston et al., 1995), or work system oriented (Denning, 1995; Buzacott, 1996) aspects of process redesign. However, existing analytical studies focus on a single aspect of process redesign, and pure case-based studies provide a general set of rules that are clearly not universally applicable.

Our research thus far (e.g., Seidmann and Sundararajan, 1996a, b) and our ongoing work have attempted to fill this gap by analyzing general multidimensional process redesign situations in a rigorous framework. Based on case studies, and our process redesign work with a Fortune 50 company over the last year, we provide a comprehensive analytical framework within which process owners and managers can study when to redesign business processes and how existing process variables affect optimal design. Many of the claims in the original process reengineering articles ignore some of the fundamental trade-offs between old and new job designs; for example, the literature is replete with examples of the successful application and the individual superiority of the case management approach or of the empowered team approach. The advantages of such teams are evident – however, loss of functional specialization and departmental control can adversely affect throughput and quality. The nature of these trade-offs is the focus of Seidmann and Sundararajan (1996b). We also study how simultaneous changes in certain key aspects of the work environment such as the automation of workflows, the introduction of incentive schemes and the elimination of hand-offs affect one another and integrate with the overall organizational structure.

In this paper, we extend this general framework by examining the effects of asymmetry on the optimal design of business processes. The first measure of asymmetry we use is task-time asymmetry – the imbalance in the size of the tasks that constitute the job within a process. Certain jobs consist of uniform chunks of work while others are comprised of tasks of highly variable duration. We show that this is a concern in process design, and that it has a significant effect on the gains from process redesign. The other measure of asymmetry we consider is information asymmetry. The effects of information asymmetry on performance-based contracts have been studied extensively by economists and management scientists. Some of the work in this area related to our analysis is discussed in the following section. However, most of these papers ignore the effect of "production" aspects of processes and its relation to information asymmetries.

The rest of this paper is organized as follows. Section 2 describes the background for our analysis. Section 3 outlines our model. Sections 4 and 5 examine the two classes of asymmetry discussed above. Section 6 concludes and offers directions for future research.

2. Background

Process redesign philosophies are most commonly communicated as a set of general case-based rules. The inefficiencies that existed in some corporations were so enormous that these general rules yielded radical improvements, irrespective of whether they were optimal or not. We briefly examine some of these rules and the effect they have on a process. We mention cases in which they have been applied and discuss related research. One of our process redesign models is then described – this forms the basis for analysis in the subsequent sections.

2.1. Workflow design

One of the most widely observed effects of process redesign is the consolidation of tasks that were performed sequentially by specialized departments into unified jobs performed by multi-functional workers or teams. The sequential system of work was passed down from the economic and manufacturing philosophies of Adam Smith and Frederick Taylor to the service organizations of today. Systems of this kind enjoyed the benefits of division of labor, described as follows by Adam Smith:

"...the division of labor, however, so far as it may be introduced, occasions, in every art, a proportionable increase in the productive powers of labor....owing to three different circumstances;
first, to the increase in dexterity in every particular workman; secondly, to the saving in time which is commonly lost in passing from one species of work to the other; and lastly, to the invention of a great number of machines which facilitate and abridge labor.\ldots\)". This quote will be referred to subsequently to indicate how these three circumstances have changed.

The specialization of labor has its drawbacks. Every time a job exchanges hands, there is a handoff delay. An expert or specialist may not easily understand the nature of what has been performed on a job by other specialized functional departments, and he spends unproductive time understanding the nature of the job at hand (a problem non-existent in manufacturing lines with completely uniform jobs, but certainly of consequence in service organizations with highly variable job dimensions). Besides, every time there is a hand-off, there is a possibility of error arising from incorrect re-keying of data or misunderstanding of the job specifics.

Today, organizations try to structure work around processes, not functions. The different tasks of a job are performed together by a versatile worker or a team. Exceptions are rerouted to experts. Just as the machines of the industrial revolution facilitated efficient assembly lines, the technology of our information revolution facilitates these flexible work systems that differ significantly from production lines. Consolidated tasks are enabled by expert systems and decision support systems; simultaneous work on a single document is facilitated by shared databases.

However, the benefits of these workflow changes should not be overestimated. Companies like Hallmark, which reduced its creative design process cycle time by half, had highly inefficient sequential systems (Hallmark had 25 hand-offs, and 90% of cycle time was queue time). The new process design may have been much superior, but that does not imply it was ideal.

Our study of business processes has led us to divide workflow variables into two different classes of parameters: the work system details – sequencing of tasks, degree of task consolidation and scheduling of jobs; and the job details – number of tasks within a job, relative size and nature of these tasks.

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### Work System Details:
- sequencing of tasks
- task consolidation
- scheduling of jobs

### Information & Technology Variables
- knowledge intensity
- information symmetry
- information sharing

### Administrative Variables
- decision rights
- performance measures
- compensation schemes

### Job Details:
- number of tasks in a job
- relative size of tasks
- nature of tasks
- degree of customization

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Fig. 1. Summary of process description.
and degree of customization. We have shown that, in many common organizational settings, consolidated systems can be inferior to sequential systems (Seidmann and Sundararajan, 1996b). In this paper, we explore the effects of asymmetry on these results.

2.2. Administrative issues

Process redesign calls for empowerment of workers and compensation based on performance rather than activity. Many companies are moving towards such variable compensation schemes. For instance, the customer operations organization of Xerox Corporation is extending its bonus plan from middle and senior management to include its entire 22,000-strong US workforce.

The issue of performance-based incentive schemes has been studied for many years. Some of the seminal papers in this area are Holmstrom (1979), Holmstrom (1982), and Jensen and Meckling (1976) - a comprehensive review of the work in this area is beyond the scope of this paper - a brief discussion is provided in Section 5. However, most work in the area tends to isolate the effect of different information symmetry situations and incentive contracts in a static organizational setting and does not consider the effects of changes in processes or work design. Some exceptions are Holmstrom and Milgrom (1987) and Itoh (1994). Holmstrom and Milgrom examine the effects of multiple performance measures (or measures on multiple tasks performed simultaneously) - however, this is distinct from the effect of combining distinct tasks, which is one of our concerns. Itoh considers delegation of tasks between multiple agents and concludes that, in the presence of moral hazard (a form of information asymmetry), a principal or firm owner prefers delegating all tasks to one agent. However, this study does not consider 'production' related parameters.

This leads us to our third class of process variables: administrative variables. These include the choice of performance measures, the degree of decentralization of decision rights and the design of compensation schemes. We do not address decision rights in detail in our research. However, the choice of compensation is a central theme of all our work in process design, as is the choice of performance measures.

2.3. Information and technology

Technological advances are what enabled process redesign. The “great number of machines which facilitate and abridge labor” described by Adam Smith have changed significantly since his time. Today’s service organizations use shared databases, expert systems and workflow automation software to improve process performance; more importantly, some work system designs are not feasible without supporting technology. Informational aspects of a process are also central to its design. Part of the role of technology is in adding to the information and knowledge of a worker. The “saving in time which is commonly lost in passing from one species of work to the other” (again, described by Smith) is countered today by a saving in time resulting from consolidating tasks that share information. The decision of whether to consolidate depends on the degree of information-sharing across tasks and the task commonalty.

Our final process variables are the information variables. These include the degree of knowledge and information required for a task, the symmetry of information between team members, the symmetry of information between workers and management and the degree of information sharing between tasks in a job.

3. Model outline

There are a variety of ways in which processes can be redesigned. Besides, a fair amount of process design is context-specific. However, one of the aims of our work is to investigate when ‘common-sense’ reengineering is good – in other words, under what circumstances the general rules outlined in the trade press really are applicable. Therefore, in our models, we capture the most common changes made when redesigning processes, and the most common characteristics of processes that are redesigned. These changes are outlined in Fig. 2.
We examine a process which consists of \( n \) tasks. Jobs arrive at a Poisson rate of \( \lambda \) jobs per unit time and the processing time of each task is assumed to be exponentially distributed with a mean time of \( 1/\mu \) per task. In the original system, these \( n \) tasks are performed sequentially. Each task is performed by a dedicated functional specialist. Each task can, therefore, be treated as an M/M/1 queue (Kleinrock, 1976). Each time the task is transferred between these specialists, there is a hand-off delay (see Fig. 2(A)). This delay represents the time spent by a specialist re-reading parts of the job requirements and trying to figure out what has already been done in the previous steps, how exceptions were handled, etc. The average time delay due to hand-offs is \( H \) time units. Also, since each job is done in parts by \( n \) individuals, there is a certain likelihood of error due to inconsistent understanding of the job requirements, or due to transmission errors. The error rate is monotone in \( n \). All completed jobs are checked for completeness at the end of the process by a supervisor. The erroneous ones are sent back for rework.

Reengineering typically recommends a shift from a serial (or functional) process design to a ‘case management’ approach (Hammer and Champy, 1993). After process redesign, we assume that all jobs are handled by \( n \) parallel agents. To allow for comparison between the systems, we maintain a constant workforce size. Each agent acts as a case manager and performs all the \( n \) tasks of each job. An equal fraction of arriving jobs is assigned to each agent, resulting in a Poisson arrival rate of \( \lambda/n \) to each of them. It is reasonable to assume that when shifting from a series of functional specialists to the case manager (or generalist) approach, one may observe some loss of processing speed due to reduced specialization. This reduction in the processing rate per task is by a factor of \( \beta \) (\( 0 < \beta \leq 1 \)). Therefore, processing time per task has a mean of \( 1/(\beta \mu) \), or each job has a mean processing time of \( n/(\beta \mu) \). Difficult jobs (exceptions) are rerouted to an expert. Since one person performs all the tasks, delays and errors due to hand-offs are eliminated.\(^1\)

Examples from the reengineering literature that describe changes like these are numerous – one of the more commonly known cases is the IBM Credit case (Hammer and Champy, 1993). In these cases, the underlying principle is shifting from a sequence of functional specialists to a team of multifunctional case managers.

A summary of notation used so far is given below:

- \( \lambda \) Arrival rate of jobs (Poisson)
- \( \mu \) Processing rate per task
- \( \beta \) Loss of specialization (\( 0 < \beta \leq 1 \))
- \( \alpha \) Percentage of exceptions (\( 0 \leq \alpha < 1 \))
- \( H \) Hand-off delay

A number of results can be deduced from this framework. For instance, when most tasks of a job require specific, or in-depth, knowledge of more than one function (a characteristic we term knowledge-intensive), combining the tasks results in a significant lowering of processing rate (\( \beta \) is lower), and at the same time one can expect that the percentage of exceptions will be higher (\( \alpha \) is greater). On the other hand, when a job requires a high degree of customization, resulting in a high variance across jobs, then more information must be shared among the experts and thus hand-off delays will increase.

\(^1\) In many cases, the agents share a common queue, and jobs are sent to available agents. However, in the absence of perfect monitoring, this leads to inter-agent gaming, which is too complicated to analyze in a multi-server queuing setting.
delays become more significant (\(H \) is higher). When tasks are tightly coupled, this increases hand-off errors and delays, whereas when tasks are independent and easily separable, hand-off effects are less pronounced. Results of some numerical analyses we have conducted are shown in Figs. 3 and 4. The curves shown in these figures are indifference curves. The region above and to the right of the curves is where process redesign (as described in the preceding paragraphs) is better; the region below the curve is where process redesign is inferior to existing process design. The curve itself joins the points where both designs are equally good (hence the name ‘indifference curve’). Here \( \rho \) refers to the average workload in the process (the average level of utilization \( \lambda/\mu \) of the workers).

In subsequent sections of this paper, this model is extended to examine task size asymmetry and information asymmetry, and their implications on process design.

4. Task size asymmetry

To enhance tractability, our prior analysis (and other comparable work; e.g. Buzacott, 1996) assumes that tasks are symmetric or uniform in size. In this section, we relax that assumption. We define a balanced system as one in which all tasks have processing times which are i.i.d. exponential with parameter \( \mu \). An imbalanced system is one in which successive tasks may have different processing rates (albeit still exponential). The degree of imbalance between two tasks is the normalized difference in their mean processing times. In order to isolate the effects of this asymmetry of task size, we normalize some other model parameters. In specific, we hold hand-off delays, percentage of exceptions and lowering of specialization at their base values, and impose a fixed compensation scheme. This normalization is summarized as Condition 1.

Condition 1: \( H = 0, \beta = 1, \alpha = 0 \).

The following simple lemma isolates the drivers of process redesign gain in a balanced job. All proofs are in Appendix A.

**Lemma 1.** In a balanced system, under Condition 1, there are no cost gains or reductions from process redesign.

This clearly indicates that in a balanced system, the cost-benefit drivers are hand-offs, rerouting and loss of specialization. Assume now that the system is imbalanced. There are gains from consolidating imbalanced tasks. Lemma 2 proves this for a specific kind of imbalance. Lemma 3 proves this for a specific kind of balanced queue, and Proposition 1 generalizes these results for process redesign.

**Lemma 2.** If a series of \( n \) uniform \( \text{M}/\text{M}/1 \) queues in tandem is perturbed in such a way that the processing rate of one of the servers decreases, while the processing rates of the remaining \( n-1 \) servers increase such that (a) the \( n-1 \) servers still have equal processing rates and (b) the total expected processing time of the system is preserved, then the total sojourn...
Lemma 3. The total sojourn time of n non-uniform M/M/1 queues in tandem is greater than the total sojourn time of n M/M/1 queues in tandem, each with a processing rate equal to the harmonic mean of the processing rates of the original n imbalanced queues.

Proposition 1. If Condition 1 holds, then in an imbalanced system:

(I) The total cost per job in a redesigned system is strictly lower.

(II) When any two tasks are combined, the resulting cost reduction is strictly increasing in their degree of imbalance.

(III) Process redesign reduces sensitivity to variation in workload. Also, the sensitivity to workload variation in an imbalanced system increases with an increase in its degree of imbalance.

What does Proposition 1 imply? Firstly, that processes which have variation in task size are good reengineering targets. Secondly, that the larger this variation, the more the reason to redesign. Thirdly, this redesigned process is more reliable – there will not be widely fluctuating performance levels at peak load and low load periods, and the process is less liable to overload and break down.

These are the direct insights; however, there is more to this result. In conjunction with Lemma 1, it shows that if a job can be broken up into uniformly sized tasks, then a serial system is potentially as good or better than a consolidated system. One might argue, however, that the elimination of handoff delays will result in gains from consolidation. Other effects such as knowledge intensity and customization also play a role in this context. We have performed extensive numerical analyses on our model to study the effect of varying its different parameters. Figs. 5 and 6 capture some of the results of these analyses.

Fig. 5 studies the interaction between knowledge intensity, imbalance and workload on the optimality of process redesign. Knowledge intensity is a measure of the level of specialized skills the tasks of a process warrant. If the tasks of a process are simple (for instance, data entry, or operating a telephone), and can be learned easily, then the knowledge intensity is low. However, if there is a high level of specialized expertise that is required to perform the tasks (for instance, if the process involves actuarial computations, or specialized surgery), obviously the knowledge intensity is high. Knowledge intensity affects the level to which processing rates are reduced due to loss in specialization when tasks are combined, and the percentage of exceptions rerouted to an expert due to the inability of a case manager to handle them. As in Figs. 3 and 4, the curves are indifference curves separating the region where process redesign is optimal (above and to the right of the curve) from the region where process redesign is suboptimal (below and to the left of the curve). We observe that with increasing knowledge intensity, process redesign becomes less desirable. However, when imbalance is greater, process redesign is optimal.
Also, when workload increases, the curve shifts toward the origin, increasing the region where process redesign is desirable – this is a result of the higher sensitivity of a sequential design to workload variation.

Fig. 6 studies the interaction between customization, task asymmetry and workload. The level of customization is a measure of the degree to which different jobs in the process are distinct. For instance, a standardized service process, where there is no customer specific information will have a low level of customization. On the other hand, a process in which the specifications of distinct customers differ widely, or there is a high level of customer-specific information that is required to complete tasks will have a high level of customization. The level of customization affects the hand-off delay prior to each task (when the level of customization is high, there are lengthy specifications that need to be reread before every task, and these special requests must be clearly communicated along with the job order; hence the hand-off delay is higher).

We observe that with increasing customization and increasing imbalance, process design becomes more desirable. Also, as in Fig. 5, the curve shifts towards the origin when workload increases.

Both these experiments are illustrations of our analytical results – that the optimality of reengineering is more likely in imbalanced systems, and that existing serial designs are more sensitive to workload increases. They also show that the effect of a decrease in knowledge intensity or increase in customization is in the same direction as in the balanced case; the effect is further compounded as imbalance increases.

5. Information and performance measurement asymmetry

Information asymmetry and its effects on managerial costs have been studied extensively by economists for many years. The area of economics that studies such asymmetries and their effect on performance control costs is known as agency theory. Some of the seminal work in this area has been done by Jensen and Meckling (1976) and Holmstrom (1979, 1982). The central issue of this theory concerns the optimal contract form for the control relationship in which one person, the principal, delegates work to another, the agent. The problem is all too common – “it exists in all organizations and in all co-operative efforts – at every level of management in firms” (Jensen and Meckling, 1976).

When the principal (or manager) can observe exactly the effort or behavior of the agent (or worker), the problem is simple. However, when the principal cannot observe this effort, contracting is difficult, as the principal has, at best, some imperfect measure of this effort (such as output or cycle time). Fixed compensation based on pre-specified effort levels becomes hazardous for the manager, because there is no precise way of confirming that the worker actually exerts these effort levels. However, output-based contracting is hazardous for the agents, as they are rewarded/punished for outcomes that are not completely under their control due to random fluctuations in job outputs, performance of co-workers and other environmental factors. Thus, the asymmetry of information leads to a control problem, the resolution of which is costly for the corporation. The costs associated with its resolution are collectively termed control costs.

The typical components that constitute these control costs are many, and the extent and relative importance of each component depends on the strategy the firm adopts to resolve this control problem. One such strategy is to introduce a monitoring mechanism to improve the information of the principal. However, such a mechanism is expensive and may not be feasible in many situations. Another strategy which partially resolves the problem is to contract on the imperfect measure of effort – however, since the agent is exposed to some risk, the principal has to pay the agent a risk premium. In addition, the principal has to monitor this measure of performance – the more frequently it is monitored or measured, the better the information of the principal; however, this form of output measurement is also not costless. Finally, the principal needs to design an optimal contract; in theory, this involves solving a complex functional optimization problem (for a precise analysis, see Holmstrom, 1979). In practice, it is a compli-
cated and expensive procedure, especially if there is a wide variety of different contracts possible for different agents. Besides, these contracts have legal and administrative costs associated with them. These design and administrative costs are a crucial and often ignored source of organizational expenditure.

In this section, we provide a brief and qualitative discussion of the impact of reengineering on these cost drivers. The following proposition summarizes the findings of this section. The subsequent paragraphs discuss qualitatively why each of the claims of the proposition is true.

**Proposition 2.** Process redesign can reduce control-related costs in three separate ways:

(I) By reducing worker-worker information asymmetry.

(II) By reducing performance measurement costs.

(III) By reducing administrative costs.

Consider part (I) of the proposition. Suppose there are $n$ workers in a sequential work system. Assume that their individual output is not measurable; the only measure that the principal has is a measure of the final output of the process. As previously stated, the workers bear the risk of mis-measurement due to the fact that the output is a random function of their effort, and need to be compensated for bearing this risk. However, in this situation, a worker also bears further risk, as the value of the output measure is determined not only by his or her own effort, but by the effort of the other workers as well. Now consider a case manager-oriented work system. True, there is still the risk associated with the randomness of the output measure; however, this output measure is determined by the effort level of that worker alone, and it is therefore a superior signal of the agent's effort. The risk premium is thereby reduced, and the control costs to the organization go down.2

If the individual output of each worker is measurable, Proposition 2 (I) is not of consequence. Consider a sequential work system in this situation. For each job, the principal measures the output of each worker; this implies that there are $n$ measurements made per job. Now examine the case management/generalist approach. For each job completed, the principal makes exactly one measurement. Therefore, if output measurement is costly, as it frequently is, control costs are reduced by process redesign; this is the result described by Proposition 2 (II).

Now consider the effect of task size asymmetry on control costs. If all tasks are uniform, and the measure of performance is cycle time, then in a sequential work system, all workers will be offered the same contract (if they have identical preferences). However, if the tasks are asymmetric, then different tasks will require different contracts – in the case of all tasks being different in size, there are $n$ contracts to design and administer. If these tasks are consolidated by using a reengineered work system, then only one contract is required, which uses the output of the entire job as its performance measure. Thus, if contract design is expensive, and if the cost of contract administration increases with contract variety, process redesign reduces control costs, which is the result summarized by Proposition 2 (III).

In the context of (III), one may argue that the nature of a contract depends not only on task size, but also on the nature of the task. However, this observation only strengthens our results. If contracts differ depending on task nature, in a sequential work system, one may require up to $n$ contracts even if task sizes are uniform. On the other hand, in the redesigned system, only one contract is required, which adds to the result in (III).

To formalize these results requires considerably more modeling and analytical machinery than the scope of this paper allows. It should be clear, however, that these three drivers of cost reduction are of significance when contemplating the relative benefits of different process designs.

6. Conclusions and ongoing research

Our research shows why no set of process reengineering rules can be universally applicable. The
optimal design of a process is dependent on the characteristics of the tasks performed within the process and the performance control measures of the organization. In this paper, we have studied the effect of task and information asymmetry on the design of a typical business process. We demonstrate how task size asymmetry increases the desirability of a typical reengineered process design, and also show that the optimal design of a process does not always lead to the well-celebrated case management approach. Our mathematical models and analytical results form a basis by which process design options can be studied. Using extensive numerical examples, we divide our parameter space into the regions where two job design options are, respectively, optimal. The managerial implications of the results we derive can be summarized as follows:

1. When a process comprises non-uniform tasks, a case manager approach is a good idea. This design is not only cost-effective, but also more robust to sudden variations in workload.

2. Increased customization, decreased task difficulty and increased task interdependence all suggest that using a generalist will be better, as discussed in (Seidmann and Sundararajan, 1996b). Moreover, the non-uniformity of tasks intensifies the case for process redesign—these characteristics reinforce each other as drivers of process redesign.

3. The functional specialist approach is well suited for a process with uniform, symmetric tasks. We also find that low task-interdependence, high knowledge intensity and low job-to-job variability further support the use of a series of functional specialists.

4. When tasks are non-uniform, and information asymmetries exist between successive workers, then process redesign complements performance based incentive compensation. There are three separate drivers of this complementarity: reduced information asymmetry, reduced performance-measurement costs, and reduced administrative costs.

Our current research is focused on examining the operational and economic implications of multiple process versions and further analyzing the precise nature of certain interactions between process design, performance control and information technology. We are also modeling diverse technology issues such as information sharing over intranets and their implications for the optimal design of work teams.

References


Appendix A

Proof of Lemma 1. The sequential system consists of \( n \) M/M/1 queues in tandem. The arrival rate at each queue is \( \lambda \); this follows from the fact that in an M/M/1 system, the departure and arrival process are identically distributed. The total sojourn time for each task is therefore exponentially distributed with parameter \( \mu - \lambda \), and the expected delay per task is \( n/(\mu - \lambda) \). The total expected delay per job is therefore \( n/(\mu - \lambda) \). In the parallel system, since the job arrivals are equally distributed between the \( n \) agents, the arrival rate to each agent is \( \lambda/n \). Since \( \beta = 1 \) and \( H = 0 \) it follows that the processing rate for each job is \( \mu/n \). The expected delay time for each job is therefore \( n/(\mu - \lambda) \), which is identical to the sequential system.

In both systems, the total compensation per job is the same. Therefore, whatever the value of \( c_p \), the total cost per job in both systems is the same.

Proof of Lemma 2. Consider \( n + 1 \) serial M/M/1 queues with processing rates \( \mu \). Increase the mean processing time of the \( n + 1 \)th of them by \( \varepsilon \) (i.e. increase one to \( (1/\mu) + n\varepsilon \) and reduce the rest to \( (1/\mu) - \varepsilon \)). The net processing time of the servers is still \( (n + 1)/\mu \). Any perturbation in the scope of the lemma can be modeled by choosing an appropriate \( \varepsilon > 0 \).

The processing rate of the first \( n \) servers is \( \mu/(1 - n\varepsilon\mu) \), and the processing rate of the \( n + 1 \)th is \( \mu/(1 + n\varepsilon\mu) \). The new total expected sojourn time is

\[
\frac{n}{\left(\mu/(1 - n\varepsilon\mu)\right) - \lambda} + \frac{1}{\left(\mu/(1 + n\varepsilon\mu)\right) - \lambda}
\]

\[
= \frac{(n + 1)(\mu - \lambda) + \lambda n \varepsilon \mu + n^2 \mu \varepsilon \lambda + 2n^2 \mu^2 \varepsilon^2 \lambda}{(\mu - \lambda - n\lambda \varepsilon \mu)(\mu - \lambda + \lambda \varepsilon \mu)}
\]

This is strictly greater than \( (n + 1)/(\mu - \lambda) \), which is the total sojourn time of the balanced system.

Proof of Lemma 3. Consider \( n \) M/M/1 queues in tandem, with processing rates \( \mu_1 \) through \( \mu_n \). Let their harmonic mean be \( \mu_h \) (\( \mu_h = n/\sum_{i=1}^{n} (1/\mu_i) \)). We wish to show that the total sojourn time of these queues is greater than the total sojourn time of \( n \) queues in tandem, each with a processing rate equal to the harmonic mean of the processing rates of the original \( n \) queues. We call this conjecture (H) and proceed to prove this by induction.

The result is trivial for \( m = 1 \). Assume it is true for \( n = m \). We have to show it is true for \( n = m + 1 \). Consider a system of \( m + 1 \) queues in tandem with corresponding processing rates \( \mu_1 \) through \( \mu_{m+1} \). Let the harmonic mean of \( \mu_1 \) through \( \mu_m \) be \( \mu_h \), and the harmonic mean of \( \mu_1 \) through \( \mu_{m+1} \) be \( \mu_{h+1} \). Without loss of generality, assume that \( \mu_{m+1} = \min \), \( \mu_i = \mu_{m+1} \leq \mu_h \).

The equality holds only if \( \mu_1 = \mu_2 = \cdots = \mu_{m+1} \), which is not true in our case. Therefore,

\[
\mu_{m+1} < \mu_{h+1} \quad (A.1)
\]

Since (H) holds for \( n = m \),

\[
\frac{1}{\mu_1 - 1} + \frac{1}{\mu_2 - 1} + \cdots + \frac{1}{\mu_m - 1} + \frac{1}{\mu_{h+1} - 1} > \frac{m}{\mu_h - 1} \quad (A.2)
\]

Now consider a system of \( m + 1 \) queues in which the first \( m \) servers have a processing rate \( \mu_h \), and
the last server has a processing rate $\mu_{m+1}$. It is easy to show from the definition of harmonic mean for $m$ and $m+1$ elements that
\[
1/\mu_{m+1} = \left(\frac{m}{m+1}\right) \frac{1}{\mu_m} + \left(\frac{1}{m+1}\right) \frac{1}{\mu_{m+1}}
\]
\[
= \frac{m\mu_{m+1} + \mu_m}{(m+1)\mu_m\mu_{m+1}}.
\]
(A.3)

Now let
\[
\varepsilon = \frac{\mu_{m+1} - \mu_m}{(m+1)\mu_m\mu_{m+1}}.
\]

It follows from A.1 and A.3 that $1/\mu_{m+1} = (1/\mu_m) + \varepsilon$ and $1/(\mu_m) = 1/(\mu_{m+1}) - \varepsilon$.

Using $\mu = \mu_{M+1}$ and $\varepsilon = (\mu_m - \mu_{m+1})/(m+1)\mu_m\mu_{m+1}$, Lemma 2 implies that
\[
\frac{m}{\mu} + \frac{1}{\mu_{m+1}} > \frac{m+1}{\mu_m}.
\]
(A.4)

Eqs. A.2 and A.4 prove (H) for $n = m + 1$, which completes the proof.

Proof of Proposition 1. Consider $n$ non-uniform tasks in series with corresponding processing rates $\mu_1$ through $\mu_n$. Call this queue system (Q1). Let $\mu_H$ be their harmonic mean.

Part (I) Consider $n$ uniform tasks in series with processing rates $\mu_1$. Call this queue system (Q2). Let $C_{Q1}$ and $C_{Q2}$ be the total costs per job of Q1 and Q2. Lemma 3 indicates that the delay time of (Q1) will be greater than that of (Q2). Since they both have the same expected processing time $\Sigma_{i=1}^n (1/\mu_i)$, they have the same compensation costs, and therefore $C_{Q1} > C_{Q2}$. Now consider consolidating both these systems into $n$ parallel queues, each with arrival rate $\lambda/n$. Call these parallel systems (P1) and (P2), respectively. Let $C_{P1}$ and $C_{P2}$ be the total cost per job of these systems. Since servers in both have processing rates of $n/\Sigma_{i=1}^n (1/\mu_i)$, these systems have the same delay time and compensation costs per job, and therefore $C_{P1} = C_{P2}$. But by Lemma 1, $C_{Q2} = C_{P2} \Rightarrow C_{Q1} > C_{P1}$, which is our desired result.

Part (II) Assume $\mu_1$ and $\mu_2$ are combined. Let $\mu_{12}$ be the harmonic mean of $\mu_1$ and $\mu_2$. It is easily shown that $1/\mu_{12} = 1/\mu_1 - \varepsilon$ and $1/\mu_2 = 1/\mu_1 + \varepsilon$, where $\varepsilon = (\mu_1 - \mu_2)/(2\mu_1\mu_2)$ (this is just a special case of some of the analysis in Lemmas 2 and 3). $\varepsilon$ is a measure of the task size imbalance. Now Lemma 2 implies that the total sojourn time of $\mu_1$ and $\mu_2$ in series is
\[
2(\mu_1 - \lambda) + 2\mu_1\varepsilon\lambda + 2\mu_1^2\varepsilon^2\lambda > \frac{2}{(\mu_1 - \lambda)}.
\]
(A.5)

Call the difference in sojourn times $\varphi$. Now,
\[
\varphi(\lambda, \mu_H) = \frac{2\mu_1\varepsilon\lambda + 2\mu_1^2\varepsilon^2\lambda}{(\mu_1 - \lambda)^2 - (\lambda\mu_H)^2}.
\]
(A.6)

It is easily seen that $\partial\varphi/\partial\varepsilon > 0$ for all $0 < \varepsilon < 1/\lambda$ and $1/\mu_H$.

Part (III) Using A.6 we calculate $\partial\varphi/\partial\lambda$ to be
\[
\frac{\partial\varphi}{\partial\lambda} = \frac{2\mu_1\varepsilon + \mu_1^2\varepsilon^2}{(\mu_1 - \lambda)^2 - (\lambda\mu_H)^2}
+ \frac{4\varepsilon(1 + \mu_1\varepsilon)(\mu_1\mu_2 + \mu_1^2\varepsilon^2)}{[(\mu_1 - \lambda)^2 - (\lambda\mu_H)^2]^2} > 0
\]
(A.7)

in this region we are interested in. This proves the first part of our desired result. Also, using A.6 and A.7, we see that
\[
\frac{\partial\varphi}{\partial\varepsilon} = \frac{\mu_1 + \mu_1^2\varepsilon^2}{(\mu_1 - \lambda)^2}.
\]
(A.8)

Since $\varphi$ is increasing in $\varepsilon$, (A.8) implies that $(\partial^2\varphi/\partial\varepsilon^2) > 0$ which is the second part of our desired result.