Dynamic Pricing of Network Goods with Boundedly Rational Consumers

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Overview

- When is unbounded rationality a good approximation?
- Our (tentative) approach to answering this question is by studying a series of examples.
  - Choose a set of 'standard' economic models in which agents have unbounded rationality (the UR models).
  - Analyze a model of the same phenomenon in which agents have bounded rationality (the BR models).
  - Compare the "output" of the two models.
- Our first example: monopoly pricing for a network good.

A model of network goods

- A monopoly firm sells a homogeneous network good (a service, rather than a durable good).
- Unit mass of a continuum of consumers, indexed by their type $\theta \in [0,1]$ drawn from a distribution with CDF F
- If the price in any period is *p*, then a consumer of type θ purchases the good in that period if

$$\theta q_E \geq p,$$

where q_E is the total demand <u>expected</u> by the consumer in that period.
- Variable cost equals zero.

A discrete-time formulation

- Suppose the firm varies its price at equally-spaced time intervals *t = 0, h, 2h, 3h,…*
  - *h* is the length of the time interval (more on this later).
- Sequence of events for a UR model
  - The firm announces its price *p(t).*
  - Each consumer forms an expectation of demand $q_E(t)$.
  - A consumer of type θ purchases if $\theta \geq \frac{p(t)}{q_E(t)}$.
  - The realized demand is $q(t) = 1 - F\left(\frac{p(t)}{q_E(t)}\right)$.
  - The firm's profit in period *t* is $p(t)q(t)$.

Outcomes in a UR model

- Since consumers are unboundedly rational, they form rational demand expectations, which are fulfilled.

$$q(t) = 1 - F\left(\frac{p(t)}{q(t)}\right).$$

- The firm sets the same price *p(t)* in each period, and demand is constant across time.
- For instance, if $F(\theta) = \theta$, then $q^{UR}(t) = \frac{2}{3}$, $p^{UR}(t) = \frac{2}{9}$
- Why the UR model seems implausible for this problem:
  - The extent of knowledge and computation that the model has consumers performing seems high (identifying other consumers' preferences, forecasting demand based on these preferences,…)
  - The predictions of the model do not appear to be consistent with observed pricing and demand patterns

Sequence of events in a BR model

- The firm announces its price *p(t).*
- Consumers who pay attention to *p(t)*:
  - Determine some subset of past demand $q(t-h), q(t-2h), \ldots$
  - Form an expectation of demand $q_E(t)$.
  - Make a decision based on the relative values of $p(t)$ and $\theta q_E(t)$.
- Consumers who do not pay attention to *p(t)* continue doing what they were doing in period (*t–h*)

Modeling bounded "cognition"

- Attention:
  - If the length of the time interval is *h*, then a fraction λh of consumers of each type pay attention to *p(t)* in period *t*, and make a decision.
- Ability to forecast:
  - Unboundedly rational: $q_E(t) = q(t)$.
  - Myopic: $q_E(t) = q(t-h)$.
  - Myopic and stubborn: $q_E(t) = \gamma q(t-h) + (1-\gamma)\omega$.

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A continuous-time approximation

- If $0 \le p(t) \le q(t)$, and under the following BR model:
  - Bounded attention: If the length of the time interval is *h*, then a fraction λh of consumers of each type actually make a decision in period *t*, and
  - Myopic forecasts: $q_E(t) = q(t-h)$,

  then the time-rate of change in demand as $h \to 0$ is:

$$q'(t) = \begin{cases} 0, \; q(t) = 0; \\ \lambda\left[1 - F\left(\frac{p(t)}{q(t)}\right) - q(t)\right], 0 < q(t) \le 1, 0 \le p(t) \le q(t); \\ -\infty, \; 0 < q(t) \le 1, \; p(t) > q(t). \end{cases}$$

  - This law of motion remains unchanged for forecasts that are "more rational" than myopic (more on this later).

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Summary of the firm's problem

Chooses the price trajectory $p(t)$

that maximizes: $\int_0^\infty e^{-rt} p(t) q(t)$

subject to the law of motion.

We can restrict our attention to stationary policies $p(t) = \alpha[q(t)]$.

The value of a policy α at an initial state *q* is:

$$V_\alpha(q) = \int_0^\infty e^{-rt} \alpha[q(t)] q(t), \;\; q(0) = q.$$

The value function at an initial state *q* is: $V(q) = \sup_\alpha V_\alpha(q)$.

A policy is optimal if its profit attains this supremum at every state *q*.

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Recall the UR model

- Under the UR model, demand in any period satisfies rational expectations: $$q = 1 - F\left(\frac{p}{q}\right).$$
- For each q, define *P(q)* implicitly as the largest solution of the above equation: $$P(q) = \max\{p : q = 1 - F(\tfrac{p}{q})\}.$$
  - (also the best "stay-where-you-are" price at *q*).
- Under the optimal rational-expectations equilibrium, demand *q* solves: $$q^{UR} = \arg\max_q [qP(q)].$$

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Results: Myopic consumers

1. The rational-expectations demand cannot be the steady state of an optimal price trajectory

- q is a steady state for the optimal policy α^* if $$q(t) = q \text{ implies that } q(s) = q \text{ for all } s > t.$$
- Theorem: The optimal rational expectations demand q^{UR} is not a steady state for the policy that this optimal for the BR model with myopic customers.

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Results: Myopic consumers

2. Solution to the optimal dynamic pricing problem – a "target policy."

- When $F(\theta) = \theta$ (uniform distribution of types), the firm's optimal pricing policy is: $$\alpha^*(q) = \begin{cases} 0, & q < \sigma^*; \\ P(q), & q = \sigma^*; \\ q, & q > \sigma^*, \end{cases}$$

  where the optimal target $\sigma^* = \dfrac{2\lambda}{3\lambda + r} < \dfrac{2}{3} = q^{UR}$.

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Results: Myopic consumers

2. Solution to the optimal dynamic pricing problem – a "target policy."

- Variation in optimal policy for
  - *F* concave
  - *F* convex

Myopic and stubborn consumers

- Attempt to see what happens when consumers are less rational than myopic.
- Consumers base their demand forecast on a weighted average of the myopic forecast and a shared *stubborn* forecast ω.

$$q_E(t) = \gamma q(t-h) + (1-\gamma)\omega$$

ω : a fixed parameter.

$\gamma = 1 \Rightarrow$ consumers are purely myopic.

$\gamma = 0 \Rightarrow$ consumers are purely stubborn.

Myopic and stubborn consumers

- Law of motion:

$$q'(t) = \begin{cases} 0, \; q(t) = 0 \\ \lambda\left[1 - F\left(\frac{p(t)}{\gamma q(t) + (1-\gamma)\omega}\right) - q(t)\right], \; 0 < q(t) \le 1, \; 0 \le p(t) \le \gamma q(t) + (1-\gamma)\omega \\ -\infty, \; 0 < q(t) \le 1, \; p(t) > \gamma q(t) + (1-\gamma)\omega \end{cases}$$

Myopic and stubborn consumers

Preliminary results

- The monopolist's optimal price trajectory is generated by a target policy with target $\sigma(\gamma, \omega)$.

Myopic and stubborn consumers

Preliminary results

- $\sigma(\gamma, \omega)$ is strictly increasing in γ, and has the following values at its end points:

$$\sigma(0, \omega) = \frac{\lambda}{2\lambda + r}.$$

$$\sigma(1, \omega) = \frac{2\lambda}{3\lambda + r}.$$

Concluding remarks

- Target policy more realistic than REE.
- Model with both myopic and UR customers.
- Concave and convex network value functions – e.g., concave network value function and uniform distribution of types.

- Competing network goods.
- Decisions based on local network structure.
- Adaptive expectations, noisy observation.