Competing in Markets with Digital Convergence

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Abstract: This paper studies competition in the presence of digital convergence, a phenomenon that has been observed in a variety of information technology industries: handheld computing, telecommunications, consumer electronics, networking, residential broadband and broadcast video, among others. Digital convergence increases the value and flexibility of products and services, but also increases the substitutability of products that were previously in distinct industries, therefore presenting a critical trade-off for managers making technological and platform scope choices. We analyze this trade-off between product value and product substitutability by developing a new model of competition in converging industries that generalizes popular models of imperfect competition, admitting endogenous product scope choices, variable substitutability across products and industry boundaries, and purchases of multiple technology products by individual consumers.

We establish four different kinds of equilibria, which characterize distinct stages of digital convergence. Our results show that early stages of convergence feature increasing prices and profitability, which eventually fall if the extent of convergence progresses beyond a critical point. However, if firms can bilaterally and strategically control the degree of convergence in their industries, their equilibrium strategies can sustain higher prices and profits even when industry boundaries blur. We also describe examples of equilibria in which consumers may buy multiple general-purpose products, using each for a specialized subset of their needs, and discuss how this may lead to cyclical demand trends between specialized and general-purpose digital products. The effect of technological changes that alter the fixed costs of expanding product scope, the variable costs of production, and the breadth of customer requirements are analyzed. Managerial guidelines based on each of our results are presented.

An extended appendix of this paper is available at

http://oz.stern.nyu.edu/papers/cmdc-appendix.pdf

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"A key to succeeding in the converged economy is resisting the temptations to enter certain markets and to know when to say no. I'm as proud of what we don't do as what we do do." (Steve Jobs, 2004).

1. Introduction

In the last few years, rapid advances in information technologies have resulted in the digitization of product technologies across a wide variety of industries, a phenomenon that has been referred to as *digital convergence* (Yoffie 1997). Digital convergence is often accompanied by a shift in product design from rigid hardware architectures supporting narrow sets of functionalities towards *platform-based* architectures characterized by the use of powerful general-purpose digital hardware, and the reliance on a software platform to implement a broad set of functionalities. While this has led to more valuable and flexible products, the use of common underlying digital technologies across industries has also resulted in overlapping sets of functionalities being provided by products in previously distinct industries. Consequently, consumers now begin to view products across previously complementary industries as imperfect substitutes (Greenstein and Khanna 1997), thereby redefining the nature of choice and competition across these industries, and the strategic focus of companies within each industry.

This tradeoff between increasing product value and increasing inter-industry substitutability is central to the economics of digital convergence, and the primary focus of our paper. It is illustrated well by a recent example. Cellular telephones in the early 1990's were limited analog voice communication devices; in contrast, the current generation of Symbian and Microsoft Smartphone platform-based mobile handsets enable powerful IP-based data communications applications, as well as personal information management, multimedia and web browsing. A similar transition has occurred in the adjacent handheld computing industry: products have evolved from the hardwired Sharp Wizard organizers to the current generation of PocketPC and PalmOS-based personal digital assistants. The latter are platform-based devices, are far more effective at personal information management tasks, provide a robust interface with PC-based applications, and also have well-developed voice and data communications capabilities.

At their core, the former set of devices are still mobile telephones and the latter set are still handheld computers. By virtue of their expanded features and versatility, devices in each industry are more valuable to consumers. However, due to the bilateral shift to a platform-based architecture and the accompanying expansion in their breadth of functionalities, converged products in the (neighboring) mobile handset and handheld computing industries are now viewed as substitutes. Moreover, the mobile telephony and computing industries are not isolated examples. Similar trends are currently prevalent in the personal computing, consumer electronics, Internet access, cable television, wireline telephony and networking equipment industries (Baker and Green 2004). As the IT component of products and services continues to increase across different industries, similar transformations are likely in other segments of the economy. To quote Intel's CEO from a recent trade-press article: "As technology converges, our opportunities expand. This is where we're putting all of our resources." (Baker and Green 2004).

In this paper, we study this trade-off between the increase in value (which we term the *value effect*) and the accompanying increase in substitutability (which we term the *substitution effect*) by developing a model of inter-industry competition between converging platform-based IT products. Our model specifically captures those effects of competition that are crucial to markets with digital convergence – consumers that demand different sets of product functionalities, products with variable product scope that have different levels of effectiveness at fulfilling these functionality needs, and firms that make strategic product scope and pricing choices.

We show that as convergence expands the scope of platform-based products bilaterally across industries, these industries evolve through three different kinds of equilibria – *local-monopoly* equilibrium, *adjacent-markets* equilibrium, and *competitive* equilibrium – which describe pricing equilibria at low, intermediate and high levels of convergence. Under the first two equilibria, the value effect of digital convergence dominates the substitution effect, and both prices and profits rise as product scope increases. Under the competitive equilibrium, however, the substitution effect dominates the value effect, and prices and profits fall bilaterally across industries. We highlight how technological changes that alter the fixed costs of increasing product scope, the breadth of customer needs and the unit cost of products affect pricing and profits under each equilibrium. Furthermore, if firms have complete strategic control over the scope of their products, we establish that any subgame-perfect outcome features relatively low levels of digital convergence, and that prices in the presence of inter-industry competition may in fact be higher than those in its absence. An appropriately chosen technology and product strategy can therefore sustain high levels of profitability even when industry boundaries threaten to blur.

We also provide illustrations of a fourth kind of equilibrium, the *non-exclusive* equilibrium, which occurs at very high levels of industry convergence. It highlights an interesting trajectory in purchasing patterns, wherein consumers initially buy multiple specialized (complementary) products at low levels of product scope, shift to buying one general-purpose device as the scope of each expands, and resume buying multiple substitutable products at very high levels of product scope.

The rest of the paper is structured as follows. Section 2 discusses existing research related to convergence and places our model of imperfect competition in the relevant literature. Section 3 outlines the model and briefly discusses some monopoly results. Section 4 characterizes the different kinds of equilibrium configurations, and analyzes symmetric convergence that is driven by exogenous factors. Section 5 describes optimal strategic choices of product convergence, establishing a family of subgame perfect equilibrium outcomes, and analyzes their sensitivity to changes in cost structure driven by technological progress. In section 6, we relax some of the assumptions of sections 3 through 5, which provides some interesting additional results. Section 7 concludes with a summary of the managerial implications of our results and a brief discussion of ongoing work. Section 8 presents an outline of the proofs of the paper's results; Section A of an extended appendix presents these proofs in detail. Section B of this extended appendix presents a base-case monopoly model in detail.

2. Related research and our modeling approach

A pioneering collection of essays in Yoffie (1997) represents a majority of existing research on product convergence and its effects on industry structure. In particular, Greenstein and Khanna (1997) predict that convergence at the product level will lead to a higher intensity of competition. In contrast, summarizing a seminar on the telephone-cable TV convergence, Katz and Woroch (1997) observe that while convergence can be a source of increased competition if it creates new entry incentives and opportunities in each others' markets, increased concentration may reduce competition if there are significant economies of scale or scope across multiple markets. None of these papers present a formal model that capture these contrasting economic effects. In a related empirical paper, Cottrel and Nault (2004) study the management of product variety in the software industry, and how this affects economies of scope. They find that firms that sell products that encapsulate more application categories perform than.

Product convergence is a relatively recent phenomenon. However, a similar transformation in manufacturing *processes* occurred in the early part of the twentieth century, during which identical production technologies such as machine tools were adopted across several different manufacturing industries. Ames and Rosenberg (1977) provide a good account of this transformation. A sizeable literature has followed them in studying the new generation of flexible manufacturing systems (Roller and Tombak 1993, Eaton and Schmitt 1994, among others). Our research differs from this stream in its level of analysis – *production-process* level versus *product* level – and in the consequent focus of analysis. The manufacturing convergence literature focuses on technology adoption patterns and on firms' product line and manufacturing strategies from the perspective of a firm *buying* and employing these converging technologies, while we focus on pricing and scope choices from the perspective of a firm *selling* converging digital products.

Analyzing these choices requires an underlying model of imperfect competition between converging industries that is rich enough to capture each variable of interest. Standard economic models of imperfect competition (Dixit and Stiglitz 1976, Salop 1979, Shaked and Sutton 1982) have been developed to analyze competition between *functionally similar* products in relatively *static* environments, and impose significant limitations on modeling digital convergence. For instance, they do not permit a clear separation between a product's (variable) endowment of functionalities and a consumer's desire for these functionalities. This places undue restrictions on the kinds of products that can be represented[2]. They also typically assume exclusionary choice – consumers are allowed to choose only one of the 'competing' products. Consequently, we have developed a new model specific to markets with digital convergence that generalizes ideas from (standard) horizontal and vertical differentiation models of competition, and explicitly reintroduces features pioneered in the characteristics approach of Lancaster (1966, 1975). The next section presents this model.

3. Model

Our model is based on an underlying *functionality space*. Heterogeneous consumer preferences are specified as subsets of this functionality space, and products are represented as (effectiveness-adjusted) bundles of functionalities. A consumer's utility from a product is derived based on the extent to which their desired functionalities are supported by a product. This approach decouples a product's endowment of features from a consumer's desire for these features, allows one to model varying "coverage" of these features by converging products, and permits non-exclusionary choices.

3.1. The basic model

The basic model is presented in four parts: the functionality space, the definition of a product, the distribution and value functions of consumers, and the production technology available to firms.

Functionality space: The basis for product design and consumer preferences is a set of *functionalities*, where a functionality is any task or activity for which a consumer can potentially utilize a product. In the context of the mobile device industry discussed earlier, mobile voice communication, streaming video, video gaming, text messaging, scheduling, and mobile spreadsheet analysis are each

[2] von Ungern-Sternberg's (1988) extension of Salop (1979) allows firms to choose their level of differentiation, but still does not permit the separation of consumer needs and product scope.

distinct functionalities. A unit circle represents the functionality space, with every point on the circle representing a distinct functionality[3]. Two functionalities that are more similar in terms of the technology needed to realize them are located closer to each other on the circle. The functionality space is assumed to be exogenous, and is not altered by the choices of the firms or consumers[4].

Products: Each product has a *core functionality*. This is the functionality it provides most effectively. In addition, each product has a level of product scope ($\frac{1}{t}$), which may be endogenously chosen by its producer, and which determines the effectiveness of a product on the other (*non-core*) functionalities. The effectiveness of a product[5] with scope $\frac{1}{t}$, on a functionality at a distance $x \in [0, \frac{1}{2}]$ from its core functionality, is $u(x) = \max\{1 - tx, 0\}$. A product's effectiveness therefore decreases as one moves away from its core functionality. An increase in a product's scope (a decrease in t) increases the product's effectiveness on all non-core functionalities, but we assume that it does not alter its effectiveness on the core functionality. The terms 'product' and 'platform' are sometimes used interchangeably, to indicate that our model could directly represent the actual functionality provided by a specific device, or indirectly represent the potential value from the platform that the device is based on.

Returning to our example, the above definition of a product implies that a mobile phone handset is most effective at providing its core functionality (voice communication), and least effective on a diametrically opposite functionality (say, mobile spreadsheet analysis), while the converse is true for a PocketPC-based computer. The effectiveness of both products on intermediate functionalities such as interactive gaming and web browsing (which require both mobile communications and an OS-based application programming interface) is somewhere in between.

Two products with diametrically opposite core functionalities are considered to be in different industries. As product scope increases, the overlap in the sets of functionalities that are supported

[3] In reality, the set of functionalities that customers value would be finite. Our model therefore represents a continuous approximation of a discrete functionality space with a large number of ordered functionalities.

[4] Changes in functionality space are briefly discussed in section 7.

[5] Many of the results hold for more general positive, non-decreasing functions of distance, rather than just the linear one used here.

effectively by each product grows, the products become closer substitutes, and the industries converge. For brevity, the product whose core functionality is located at z on the unit circle is referred to as 'the product located at z'. To ensure that every product has non-zero effectiveness on every functionality, we assume that $1 - \frac{t}{2} \geq 0$ (or $t \leq 2$)[6].

Consumers: Each consumer values a subset of functionalities in the functionality space; we assume that these subsets are continuous and of equal size, and that the consumer values all functionalities within this set equally. However, different consumers require different sets of functionalities. Consequently, each consumer is represented as an arc on the functionality space, these arcs are of constant *breadth* $r \in (0, 1]$, and are uniformly distributed around the unit circle. The consumer whose arc of functionality requirements is centered at $y \in [0, 1]$ on the unit circle is referred to as 'the consumer located at y'. The density of consumers on the unit circle is $\frac{n}{2}$.

Following Lancaster (1966, 1975), consumer utility is assumed to be additive over the set of functionalities. For each functionality, consumer utility is the product of the level of effectiveness that a product provides on that functionality and the value (0 or 1) that the consumer places on that functionality[7]. The value a consumer located at y derives from a product with slope $\frac{1}{t}$ is therefore computed by summing the effectiveness of the product over the set of functionalities that the consumer values:

$$U(y, t) = \int_{y-\frac{r}{2}}^{y+\frac{r}{2}} u(|x - z|)dx, \tag{3.1}$$

measured in monetary units. This is illustrated graphically in Figure 3.1. Note that in this figure, we have unfurled the circle and depicted it along the base of a rectangle. Moreover, the first half of the circle $[0, \frac{1}{2}]$ is replicated as $[1, \frac{3}{2}]$. This is both for clarity of illustration and for analytical convenience. The vertical dimension of the rectangle depicts the effectiveness of a product on each functionality.

Technology: All firms have identical costs of production $C(q, t) = cq + F(t)$, where q is the

[6] An example of very narrow-scope products is discussed in section 6.2.

[7] This utility specification gives our representation of consumer types a combination of taste based heterogeneity (like the types in spatial models) and heterogeneity in their valuation of quality (effectiveness). Analogously, the specification of a product also combines notions of both horizontal and vertical differentiation.

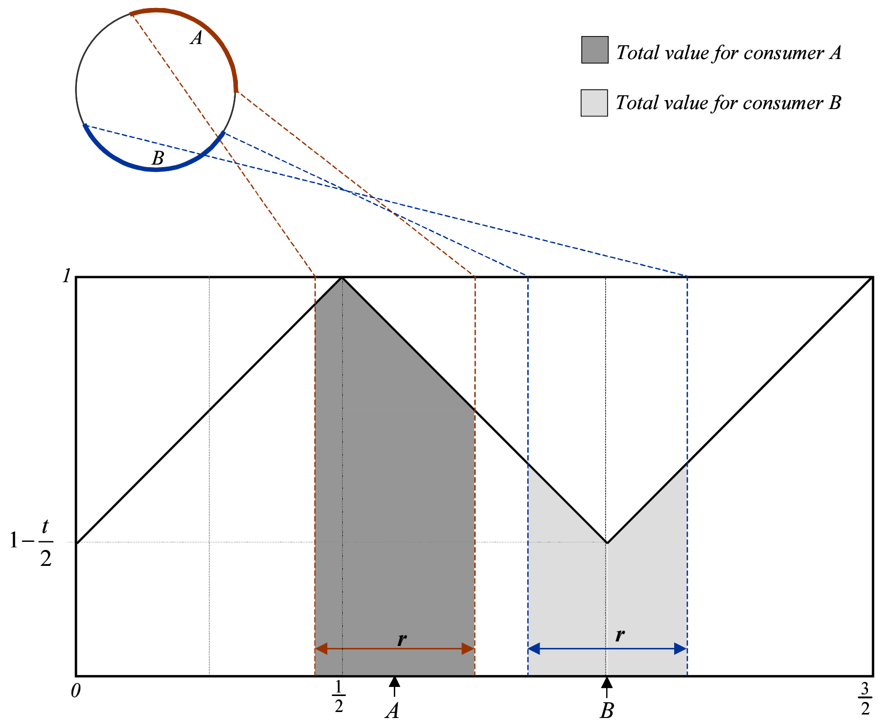

Figure 3.1: Graphically illustrates the value obtained by two consumers located at A and B, for a product whose core functionality is located at $\frac{1}{2}$.

quantity produced, and t is the reciprocal of product scope[8]. This separable form implies the following assumptions about $C(q,t)$.

1. Marginal cost of production is non-negative and constant: $C_1(q,t) = c \geq 0$, $C_{11}(q,t) = 0$.

2. Variable cost is independent of scope $C_{12}(q,t) = 0$.

In addition, the cost function is assumed to have the following properties:

3. Fixed cost is non-decreasing and convex in scope: $F_1(t) \leq 0, F_{11}(t) > 0$.

4. Marginal cost is not too high: for $t \leq 2$, $c < U(y,t)$ for all y.

Numbered subscripts to functions represent partial derivatives with respect to the corresponding argument. Constant marginal costs and increasing convex fixed costs reflect the fact that increasing the scope of a technology product is typically achieved through a superior hardware design, an increase in the functionality of the software platform, or a combination of the two, both of which increased fixed

[8] Different cost functions for different firms will result in asymmetric equilibria (rather than the symmetric equilibria we largely focus on in sections 4, 5 and 6), but will not otherwise affect the qualitative nature of the results.

Product-related	
t	Reciprocal of product scope. As t increases, scope $(1/t)$ decreases.
$u(x)$	Effectiveness of product on a functionality that is at a distance x from the core functionality
$F(t)$	Fixed cost at a level of scope $\frac{1}{t}$
c	Constant marginal cost
i, j	Indices for competing products. $i, j \in \{A, B\}$
Market-related	
$\frac{n}{2}$	Market size (density)
r	Consumers' breadth of functionality requirements
$U(.)$	Consumers' value function (willingness-to-pay)
Choice and outcome	
q	Variable representing normalized quantity. At a value q, actual demand is nq
$P^M(q, t)$	Monopoly inverse demand curve at a level of scope $(1/t)$
$P^C(q_i, t_i, t_j, p_j)$	Competitive inverse demand curve for firm i
$\pi^i(q_i, t_i, t_j, p_j)$	Gross profit (before accounting for fixed costs) for firm i
$\Pi^i(t_i, t_j)$	Net profits (after accounting for fixed costs)

Table 3.1: Summary of some of the notation used in the paper

costs, but do not substantially alter variable production costs. A partial summary of key notation (some of which is defined later in the paper) is provided in Table 3.1.

3.2. The value function, demand and some monopoly results

This subsection formalizes consumer value and demand implied by the model, and briefly describes some monopoly results, the details of which are in the paper's extended appendix. Without loss of generality, fix the location of a product at $\frac{1}{2}$, dividing consumers into two identical and symmetric segments $[0, \frac{1}{2}]$ and $[\frac{1}{2}, 1]$. At any price, demand from each segment will be identical, and we can focus our analysis on one of these segments $[\frac{1}{2}, 1]$.

In sections 3.2 through 5, the set of functionalities demanded by consumers is assumed to be not very large – more precisely, $r \leq \frac{1}{2}$. The case of $r > \frac{1}{2}$ is discussed in section 6.

Computing the value function $U(y, t)$ in the interval $y \in [\frac{1}{2}, 1]$ yields the following piece-wise continuous form:

$$U(y,t) = \begin{cases} r - t\left[\frac{1}{2}[\frac{1+r}{2} - y]^2 + \frac{1}{2}[y - \frac{1-r}{2}]^2\right], & \frac{1}{2} \le y \le \frac{1+r}{2}; \\ r - t\left[\frac{1}{2}[y - \frac{1-r}{2}]^2 - \frac{1}{2}[y - \frac{1+r}{2}]^2\right], & \frac{1+r}{2} \le y \le \frac{2-r}{2}; \\ r - t\left[\frac{1}{4} - \frac{1}{2}[y - \frac{1+r}{2}]^2 - \frac{1}{2}[1 - [y - \frac{1-r}{2}]]^2\right], & \frac{2-r}{2} \le y \le 1; \end{cases} \tag{3.2}$$

The following lemma describes some useful properties of $U(y,t)$.

Lemma 1. *For all* $y \in [\frac{1}{2}, 1], t \in [0, 2]$

(a) $U(y,t)$ *is continuous and decreasing in both* y *and* t

(b) $U_1(y,t)$ *is continuous and is piece-wise differentiable in both* y *and* t.

(c) $U_2(y,t)$ *is continuous and decreasing in* y.

Part (a) of the lemma establishes that consumers located closer to the product derive more value from it, and an increase in scope increases value for all consumers. Part (c) establishes that the gross value from a product increases faster with product scope for those consumers who are farther away from it. An increase in product scope therefore increases value, but simultaneously reduces the heterogeneity in product value; this is consistent with the trade-off between the value effect and the substitution effect of digital convergence.

The value function $U(y,t)$ determines the *inverse demand function* $P^M(q,t) = U(q + \frac{1}{2}, t)$ faced by a monopolist[9]. At a price $p = U(\hat{y}, t)$, where $\hat{y} \in [\frac{1}{2}, 1]$, all consumers located between $\frac{1}{2}$ and $\hat{y}$ buy the product, i.e. $q = \hat{y} - \frac{1}{2}$ implying a demand of $[\hat{y} - \frac{1}{2}]\frac{n}{2}$ from consumers located in $[\frac{1}{2}, 1]$, an identical demand from consumers located in segment $[0, \frac{1}{2}]$, and thus a total demand of $n[\hat{y} - \frac{1}{2}]$. As a benchmark, the optimal monopoly choices of price and scope are derived in Section B of the extended appendix. At relatively low levels of product scope ($\frac{1}{t} < \frac{r[2-\sqrt{2r}]}{2[r-c]}$), the monopolist prices such that a part of the market is excluded; however, at higher levels of product scope ($\frac{1}{t} \ge \frac{r[2-\sqrt{2r}]}{2[r-c]}$), the monopolist prices to sell to the entire market even in the absence of any regulatory mandate to do

[9] Note that the inverse demand function is $P^M(q,t) = U(y,t) = U(q + \frac{1}{2}, t)$ and not $U(q,t)$ since $q \in [0, \frac{1}{2}]$ while $y \in [\frac{1}{2}, 1]$.

so. This may have implications in industries where universal access is a social priority, as discussed briefly in that section.

4. Competition between converging products

The next three sections model competition between two firms A and B, whose products' core functionalities are exogenously diametrically opposite each other, at $\frac{1}{2}$ and 1 respectively. This models two firms in related but initially distinct industries, and that begin to compete more intensively as their industries converge as a result of bilateral increases in product scope. Competition is modeled as a two stage game with firms simultaneously choosing product scopes in the first stage, before simultaneously choosing prices in the second. For analytical convenience, the demand of firm i is represented as nq_i, and characterized in terms of the normalized parameter $q_i \in [0, \frac{1}{2}]$, which is simply demand divided by n.

4.1. Duopoly demand and profit functions

We denote a firm's own choices using the subscript i, and those of its opponent with the subscript j. Given choices of scope t_A, t_B, and the opponent price p_j, the inverse demand curve of duopolist i, denoted $P_i(q_i, t_i, t_j, p_j)$, is composed of two functional forms – the *monopoly* inverse demand function $P^M(q, t_i) = U(q + \frac{1}{2}, t_i)$, and the *competitive* inverse demand function $P^C(q, t_i, t_j, p_j)$:

$$P^C(q, t_i, t_j, p_j) = U(q + \frac{1}{2}, t_i) - U(1 - q, t_j) + p_j. \tag{4.1}$$

The economic intuition that drives the equation above is as follows: At a fixed level of product scope, duopoly demand is determined completely by a firm's price and the surplus consumers get from its opponent's product. For a sufficiently high opponent price p_j, those consumers close enough to firm i get no net surplus from firm j's product, and their demand is therefore represented by the monopoly inverse demand curve for firm i. However, those consumers close enough to product j receive a positive net surplus from purchasing product j, and will purchase product i only if it provides them with at

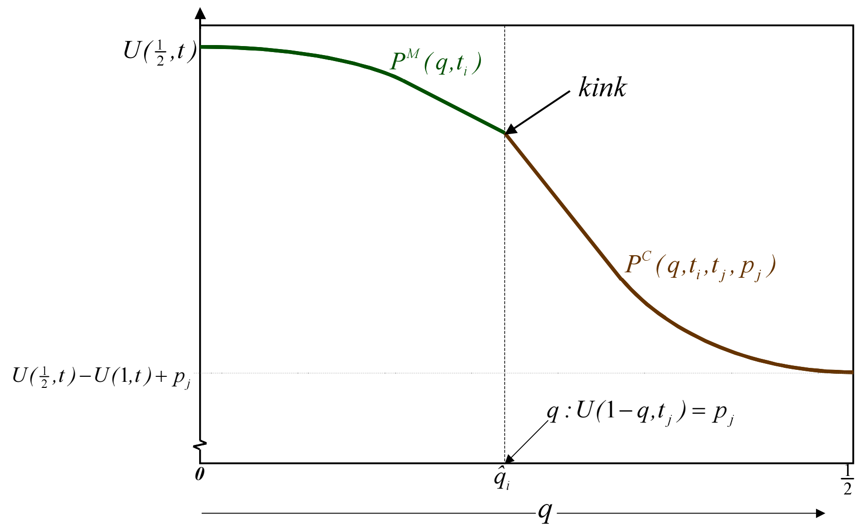

Figure 4.1: Illustrates the duopoly inverse demand curve. $\hat{q}_i = 1 - U^{-1}(p_j, t_j)$ corresponds to the indifferent consumer who gets exactly zero net surplus from product j at price p_j. All consumers to the left of this indifferent consumer have a reservation value of zero for product j and hence form the 'monopoly region' of the duopolist i's inverse demand curve, while consumers to the right of the indifferent consumer get positive net surplus, forming the 'competitive region'. The inverse demand function changes slope discontinuously at $\hat{q}_i$ and therefore has a kink, i.e. it is continuous but not differentiable, at this point.

least an equivalent level of surplus. The competitive inverse demand function in (4.1) captures this additional aspect of demand from these consumers. This leads to the following specification for a duopolist's inverse demand curve.

$$P_i(q_i, t_i, t_j, p_j) = \begin{cases} P^M(q_i, t_i), & 0 \leq q_i \leq [1 - U^{-1}(p_j, t_j)]; \\ P^C(q_i, t_i, t_j, p_j), & [1 - U^{-1}(p_j, t_j)] \leq q_i \leq \frac{1}{2} \end{cases}$$

The inverse of U is with respect to its first argument (i.e. $U(U^{-1}(y,t),t) = y$). Figure 4.1 illustrates this demand curve. The inverse demand function has a kink at $\hat{q}_i = [1 - U^{-1}(p_j, t_j)]$, where it's slope changes discontinuously, and which represents a critical transition point beyond which the intensity of competition between the industries is substantially higher. The size of the monopoly and competitive regions for firm i therefore depends on its competitor's price p_j.

The gross profit function for firm i, which represents payoffs after accounting for the variable cost of producing the product, but before accounting for the fixed cost of product scope, is

$$\pi^i(q_i, t_i, t_j, p_j) = nq_i[P_i(q_i, t_i, t_j, p_j) - c], \quad i = A, B, \tag{4.2}$$

and has the following property:

Lemma 2. *The function $\pi^i(q_i, t_i, t_j, p_j)$ has either no interior maximum, or a unique interior maximum in q_i.*

We refer to the portion of the gross profit function which corresponds to the monopoly region of $P_i(q_i, t_i, t_j, p_j)$ as the *monopoly portion*, and that which corresponds to the competitive region of $P_i(q_i, t_i, t_j, p_j)$ as the *competitive portion* of the duopolist's gross profit function.

4.2. Equilibrium pricing

Depending on the values of product scope $1/t_A$, $1/t_B$, four different equilibrium configurations are possible for the second-stage pricing game.

1. **Local-monopoly equilibrium:** Under this equilibrium, each firm sets a price in the monopoly region of their demand curves. A subset of consumers do not purchase either product.

2. **Adjacent-markets equilibrium**: Under this equilibrium, each firm sets a price *at the kink* between the monopoly and competitive regions of its demand curves, and the market is fully covered. It may seem as if this equilibrium configuration is a knife's-edge case, but it is actually feasible across a significant range of product scope values and demand parameters.

3. **Competitive equilibrium**: This is analogous to the 'standard' equilibrium that one encounters in spatial models. Firms set prices in the competitive region of their demand curves and potentially compete for each others' marginal consumers. In equilibrium, each consumer buys a single product.

4. **Non-exclusive equilibrium**: This equilibrium configuration involves multiple purchases by a subset of consumers. This occurs only when $r > \frac{1}{2}$, and is discussed in section 6.

When $r < \frac{1}{2}$, no more than one of these configurations results for any given values of product scope

$(1/t_A, 1/t_B)$. The exact product scope ranges, and a more mathematically precise statement of the following proposition are provided in Section A.4 of the extended appendix.

Proposition 1. *For each pair of feasible values of product scope* $(1/t_A, 1/t_B)$ *for the competing products, there is a unique feasible equilibrium. As illustrated in Figure 4.2.*

(a) At low levels of product scope, the outcome is a local-monopoly equilibrium.

(b) At intermediate levels of product scope, the outcome is an adjacent-markets equilibrium.

(c) At higher levels of product scope, the outcome is a competitive equilibrium.

Part (a) of the proposition indicates that for a fairly significant range of overlap in product functionality, there ends up being no real strategic interaction between the prices of the two firms; each firm behaves like a monopolist, and a portion of the market is left unserved by either firm. This local-monopoly equilibrium persists for all pairs of scope values below the curve AM in Figure 4.2.

At any pair of scope values on the curve AM in Figure 4.2, all consumers purchase one or the other product. When product scope increases beyond this point, the resulting equilibrium configuration is an *adjacent-markets* equilibrium. The market is fully covered, yet neither firm finds it profitable to try and gain market share. Intuitively, this is because the benefit of capturing an opponent's consumer by dropping prices is outweighed by the corresponding loss in profit from one's existing customer base[10]. This behavior persists across a non-trivial range of values for product scope, between AM and CA.

Beyond CA, the outcome transitions to a competitive equilibrium configuration. Product scopes are high enough to make it profitable for each firm to unilaterally increase demand at the margin, even after accounting for the losses in profits from one's own customer base. Consequently, there is downward pressure on prices, which intensifies as product scope increases further.

[10] Some readers may have noted the similarity between our local monopoly and adjacent markets equilibria, and the corresponding local monopoly and kinked outcomes in Salop (1979). While the outcomes are indeed similar, their drivers are quite different. Salop's local monopoly and kinked equilibrium configurations arise due to differences in average costs. In our model, on the other hand, they arise primarily from the higher heterogeneity in consumer valuations at low product scopes that makes it unattractive for firms to increase demand at the 'kink'. Matutes and Regibeau (1988) have a similar evolution of equilibria between competing pairs of complementary products, where over an intermediate range of reservation values, firms producing incompatible substitutable systems engage in limit pricing in what they also term an 'adjacent markets' equilibrium.

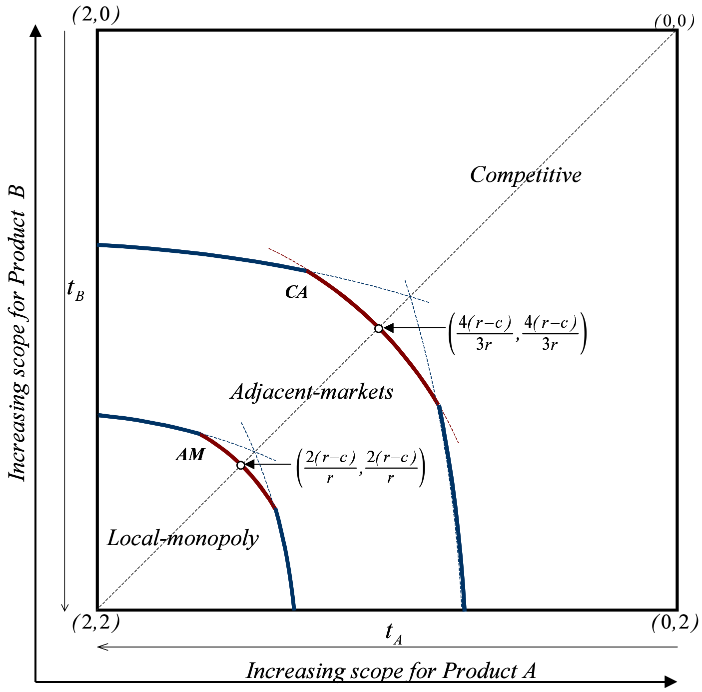

Figure 4.2: Illustrates the regions of the (t_A, t_B) space in which different equilibrium configurations occur, showing how as scope increases, the feasible configuration shifts from local-monopoly, to adjacent-markets, to competitive. The axes in the figure denote product scope. The origin corresponds to the lowest level of product scope required ($t = 2$) for each product to provide a positive level of effectiveness on the entire set of functionalities. The diagonally opposite vertex represents levels of product scope, where each product provides the entire set of functionalities at the highest level of effectiveness ($t = 0$). At this maximal level of product scope, the two products are effectively identical since each is equally effective on the entire set of functionalities.

4.3. Symmetric equilibria with exogenous digital convergence

This section derives the Nash equilibrium prices for symmetric exogenously specified product scope $t_A = t_B = t$. This corresponds to a scenario where the extent of digital convergence is not controlled by the firms, but is determined by industry-specific factors – for instance, due to progress in technology in an upstream industry (microprocessors and semiconductors in the case of computing and electronics devices), progress in a downstream industry (personal computers in the case of operating systems or application software), or exogenously specified *dejure* industry standards. Product scope could also be driven by competitive pressure from within one's industry, and our results might have some implications for this scenario, though the generality of this interpretation is limited by the fact that we do not explicitly model intra-industry competition.

Range of scope values	**Equilibrium**	**Demand**	**Price**	**Gross Profit**
$\frac{1}{2} \le \frac{1}{t} \le \frac{r^2}{r-c}$	Local-monopoly	$n\left[\sqrt{\frac{4[r-c]-r^2t}{12t}}\right]$	$\left(\frac{2r+c}{3} - \frac{r^2t}{6}\right)$	$\frac{n}{6}\sqrt{\frac{[4[r-c]-r^2t]^3}{12t}}$
$\frac{r^2}{r-c} \le \frac{1}{t} \le \frac{r}{2[r-c]}$	Local-monopoly	$\frac{n[r-c]}{2rt}$	$\frac{r+c}{2}$	$\frac{n[r-c]^2}{4rt}$
$\frac{r}{2[r-c]} \le \frac{1}{t} \le \frac{3}{4}[\frac{r}{r-c}]$	Adjacent-markets	$\frac{n}{4}$	$r[1-\frac{t}{4}]$	$n[\frac{r-c}{4} - \frac{rt}{16}]$
$\frac{3}{4}[\frac{r}{r-c}] \le \frac{1}{t}$	Competitive	$\frac{n}{4}$	$c + \frac{rt}{2}$	$\frac{nrt}{8}$

Table 4.1: Equilibria with symmetric values of product scope $1/t$ (scope increases from rows 1 to 4)

The symmetric equilibria are those along the diagonal in Figure 4.2; the corresponding prices, demand and profits are given in Table 4.1, and their characteristics are summarized in the following proposition.

Proposition 2. *At any symmetric level of product scope, there is a unique and symmetric Nash equilibrium pricing strategy. The equilibrium prices chosen by the firms are summarized in Table 4.1. As the two products converge due to an increase in scope* ($\frac{1}{t}$)*:*

(a) The price equilibrium transitions from local-monopoly to adjacent-markets to competitive equilibrium.

(b) Prices and gross profits are non-decreasing (and generally increasing) in scope at low and intermediate levels of scope, but decrease progressively at high levels of scope.

Proposition 2 is illustrated in Figures 4.3 and 4.4, which also depict the monopoly price and profit results (derived in the extended appendix) for comparison. At lower levels of product scope, there is a relatively low degree of convergence in the market (relatively low, but non-zero, overlap in the set of functionalities supported by the two products), while at the extreme right, there is complete convergence (the two products are effectively identical).

As illustrated in these figures, exogenous digital convergence initially increases both prices and gross profits. This increase is driven first by the absence of a net substitution effect for $\frac{1}{2} \le \frac{1}{t} \le \frac{r}{2[r-c]}$ under the local-monopoly equilibrium, and the price increases are accompanied by a steady increase in demand for both products. Further convergence transitions the outcome to an adjacent-markets equilibrium for $\frac{r}{2[r-c]} \le \frac{1}{t} \le \frac{3r}{4[r-c]}$. There is now genuine strategic interaction between the

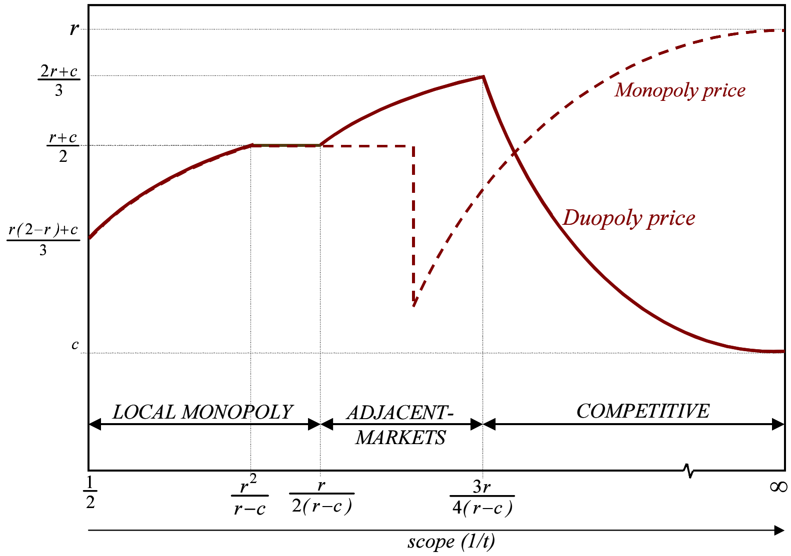

Figure 4.3: Equilibrium duopoly price as a function of scope $\frac{1}{t}$ for symmetric scope. Product scope increases along the x-axis from a level at which each product barely covers all the functionalities, to a level at which each product provides the entire set of functionalities at the maximum level of effectiveness.

firm's choices of prices; however, the value effect continues to dominate the substitution effect, and convergence continues to bilaterally increase firm profits. Despite the converging products becoming increasingly *less* differentiated, prices *rise* rather than fall, even though the market is fully covered and there are no gaps in consumer demand that insulate the firms from competition. Strikingly, the duopoly prices in this region are even *higher* than the corresponding monopoly prices (the dotted line in Figure 4.3).

The economic intuition for this unusual result is as follows: the adjacent-markets equilibrium occurs in a transition region between monopoly and standard imperfect competition. At this equilibrium, the slope on the monopoly portion of the profit function is positive, which would induce a demand increase by an unconstrained monopolist. However, the presence of the rival duopoly firm makes this demand increase impossible without reducing profits – due to the discontinuous change in the slope of inverse demand at the kink, the slope of the competitive portion of the profit function is still negative. The equilibrium response to this tension ends up being an increase in price (which keeps firms at the boundary of the local-monopoly region of the demand curve), rather than in realized demand (which

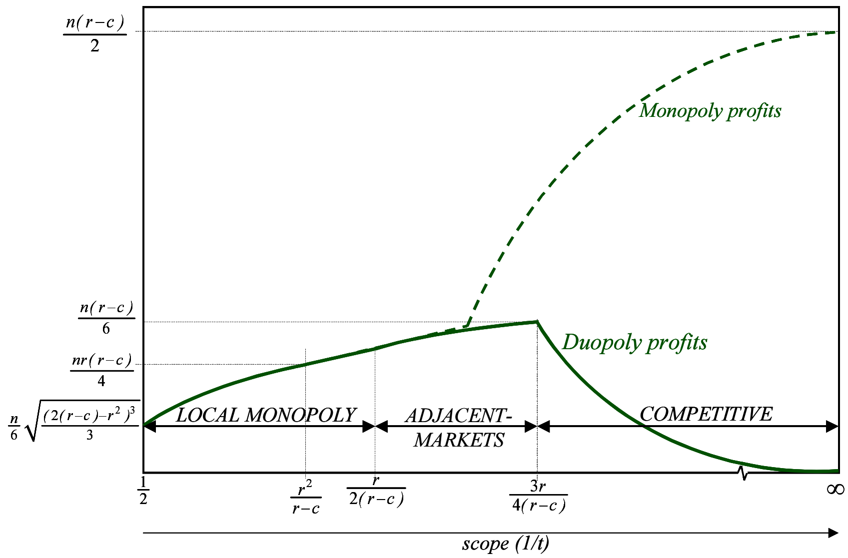

Figure 4.4: Equilibrium duopoly profits as a function of scope $\frac{1}{t}$ for symmetric scope. Product scope increases along the x-axis from a level at which each product barely covers all the functionalities, to a level at which each product provides the entire set of functionalities at the maximum level of effectiveness.

would move them to the competitive region). This also explains why even though duopoly prices are higher, duopoly profits are lower than monopoly profits.

As the products converge further, the market transitions to a competitive equilibrium, which is similar (though not identical) to that of the standard 'circle model' of horizontally differentiated duopoly. The firms split the market equally. As product scope increases, prices and profits fall. When scope is infinite ($t = 0$), the two products are identical, prices converge to marginal cost and gross profits converge to zero, as predicted by the standard Bertrand model of duopoly.

An important strategic consequence of Proposition 2 is that the benefits of digital convergence depend critically on how far convergence has progressed. The appropriate strategic response to early-stage exogenous convergence is an increase in prices despite an increase in product similarity, since the value effect is dominant, and convergence benefits firms in both industries. On the other hand, at a more advanced stage of digital convergence, the substitution effect is dominant, and convergence should be resisted to the extent possible since it is bilaterally profit-reducing. This trend is illustrated by the mobile telephony and handheld computing industries. Following the launch of mainstream converged devices from both industries – the Palm-based Handspring (now PalmOne) Treo, and the Symbian-based Nokia Communicator – average prices for these devices were actually higher than those that prevailed when the flagship products from each of these device makers was more specialized. This is despite an evident increase in the overlap in their functionality; however, these prices are coming under increasing pressure as technological progress coupled with intense intra-industry competition drives outcomes to the point where products in the two industries are highly converged. Additionally, it may be strategically sensible to exit from industries that have converged too extensively with neighboring ones, since their profit potential is likely to have eroded significantly; as the adoption of IP-based telephony increases, the wireline long-distance industry may soon be one such example.

Changes in breadth of functionality requirements: The price expressions in Table 4.1 indicate that prices always *rise* as the breadth of functionality requirements of the consumers (r) *increases*. This is interesting, because an increase in r has two economic effects, both analogous to an increase in scope $\frac{1}{t}$ – it increases the gross value of each product for all consumers, but simultaneously reduces the effective level of product differentiation. Table 4.1 indicates that the former effect dominates the latter under all equilibrium outcomes[11]. From a managerial perspective, this highlights the benefits of inter-industry actions that increase the span of the average customer's needs, even if these reduce the effective level of product differentiation, since the value effect of this kind of expansion dominates the substitution effect, independent of the extent of convergence between the industries.

Changes in marginal cost: Under the competitive equilibrium, any reduction in cost c translates into an equivalent price reduction, reflecting the high degree of competition. Under the local-monopoly equilibrium, cost decreases are shared by the firm and its consumers, and are accompanied by an increase in demand, which is consistent with the monopolistic nature of this outcome. However, under the adjacent-markets equilibrium, the entire benefit of a decrease in costs is captured by the duopolists, and prices remain unchanged[12]. This has important implications for firms' incentives to

[11] This is in contrast with the effect of an increase in product scope under competitive equilibrium. It highlights the importance of separating changes in the level of product differentiation that result from product scope choices, from those that result from changes in consumer needs.

[12] Beyond these direct effects within an equilibrium configuration, changes in r and c also affect the relative sizes of

invest in technological progress that reduces their variable product costs. The benefit that a firm realizes from these investments is likely to be highest at intermediate stages of convergence (when prices are also at their maximum). In contrast, when industry convergence has progressed to the point where firms perceive active price competition from their neighboring industries, the benefits from reductions in variable costs will flow entirely to consumers.

5. Strategic product convergence

This section establishes that when firms can strategically control digital convergence through endogenous choices of product scope, their subgame perfect equilibrium choices result in relatively low levels of convergence. Specifically, the equilibrium of the second-stage pricing subgame always a local-monopoly or an adjacent-markets equilibrium. We also examine how the extent of endogenous convergence varies as technological progress alters the fixed costs of increasing scope and the variable costs of production.

The timeline is as follows. In stage 1, firms simultaneously choose product scope $(1/t_A, 1/t_B)$. In stage 2, with perfect information about the first period choices, firms simultaneously choose prices. Denote the equilibrium second-stage choice of price by firm i as $P_i^*(t_i, t_j)$. Firm i's first stage payoff function, or its net profit function after accounting for the cost of scope and the corresponding second-stage equilibrium choices of price and quantity, is:

$$\Pi^i(t_i, t_j) = \pi^i(q_i^*(t_i, t_j), t_i, t_j, P_j^*(t_j, t_i)) - F(t_i),$$

where $\pi^i(q_i, t_i, t_j, p_j)$ is the firm's gross profit defined in (4.2), its payoff function for the second-stage pricing subgame. As established in Section 4, there are three potential equilibrium configurations in this subgame. Since the firms' first stage best-response functions are neither continuous nor monotonic[13], we adopt an indirect approach to characterize the equilibria of the game, examining ranges

the regions of t under which each equilibrium is feasible. Since $(\frac{r-c}{r})$ is increasing in r and decreasing in c, an increase in the breadth of functionality requirements, or a reduction in unit costs both increase competitive intensity by decreasing the range of scope values at which the local monopoly and kinked equilibria are feasible, and increasing the ranges at which the outcome is a competitive equilibrium.

[13] A further complication is that there are portions of the (t_A, t_B) space (where the values of t_A and t_B are substantially

of product scope that correspond to each of the second stage subgames, and using the monotonicity of gross profits to derive the subgame perfect equilibrium. The following result establishes that within any second-stage subgame, the marginal effect of changing t_i on firm payoffs is always strictly monotonic.

Lemma 3. *Given any pair (t_A, t_B) for which $P_i^*(t_i, t_j)$, $i = A, B$ exists, if (t_A, t_B) is such that small unilateral changes in product scope do not change the equilibrium configuration of the second-stage subgame, then:*

(a) If this subgame corresponds to a local-monopoly or an adjacent-markets equilibrium, each firm's gross profits are increasing in its own product scope. That is, $\frac{d}{dt_i}\pi^i(q_i^*(t_i, t_j), t_i, t_j, P_j^*(t_j, t_i)) < 0$.

(b) If the subgame corresponds to a competitive equilibrium, each firm's gross profits are strictly decreasing in its own product scope. That is, $\frac{d}{dt_i}\pi^i(q_i^*(t_i, t_j), t_i, t_j, P_j^*(t_j, t_i)) > 0$.

The derivatives in Lemma 3 are total derivatives[14], including adjustments for the equilibrium second-stage changes in $q_i^*(t_i, t_j)$ and $P_j^*(t_j, t_i)$ that accompany changes in t_i. As an immediate consequence of Lemma 3(b), since $F_1(t) \leq 0$, a pair (t_A, t_B) that results in a competitive equilibrium configuration in the second stage can never be a first-stage equilibrium choice (since either firm can reduce scope by increasing t_i, thereby increasing gross profits and reducing fixed costs). This is formalized in the next proposition,

Proposition 3. *(a) Any value of product scope $(1/t_d^*)$ that satisfies the following conditions is a*

different) in which no pure strategy equilibria may exist. It is possible that mixed-strategy equilibria do exist. However, since our payoff functions are not quasi-concave, the existence of mixed strategy equilibria is not guaranteed (Dasgupta and Maskin 1986) and their derivation is beyond the scope of this paper. A simple way around this problem (used by Economides 1984, for instance) is to restrict the first-stage action spaces of the firms to those values of scope for which second-stage pure strategy pricing equilibria exist. This is done by setting the payoffs of both firms to zero for values of (t_A, t_B) for which a pure strategy price equilibrium does not exist. Note that pure-strategy equilibria always exist for all symmetric pairs $t_A = t_B$ (and for pairs in their immediate neighborhood), and these are the equilibria we focus on.

[14] Although directionally similar, the result of Lemma 3 is distinct from that of Proposition 2, which describes the comparative statics of changing product scope symmetrically for both firms. Lemma 3 establishes the effect of changing a firm's own product scope while holding the (possibly different) scope level of its opponent constant.

symmetric first-stage equilibrium choice of scope:

$$\Pi_1^i(t_i, t)|_{t_i=t^+} \leq 0, \quad (5.1)$$

$$\Pi_1^i(t_i, t)|_{t_i=t^-} \geq 0, \text{ and} \quad (5.2)$$

$$\Pi_{11}^i(t_i, t)|_{t_i=t_d^*} < 0. \quad (5.3)$$

(b) For any non-negative fixed cost of scope $F(t) \geq 0$, the equilibrium strategic choices of product scope result in a relatively low level of convergence. Specifically, equilibrium choices of (t_A, t_B) always result in second-stage outcomes that are either local-monopoly or adjacent-markets equilibria.

Note that only (5.1) and (5.2) are necessary conditions, and when $\Pi_1^i(t_i, t)$ is continuous at $t_i = t$, reduce to the more familiar first order condition $\Pi_1^i(t, t) = 0$.The special case where increases in scope are costless i.e. where fixed cost is independent of scope ($F_1(t) = 0$) is characterized in Proposition 4.

Proposition 4. *When changing product scope is costless, i.e. $F_1(t) = 0$:*

(a) There is potentially a continuum of asymmetric subgame-perfect Nash equilibria. The first-stage choices under these equilibria correspond to those points on the line CA in Figure 4.2 (which defines the boundary between the adjacent-markets and competitive equilibrium regions) for which a second-stage pure-strategy price equilibrium exists.

(b) There is always a unique symmetric subgame-perfect Nash equilibrium. The two firms choose symmetric levels of product scope $\frac{1}{t^*} = \frac{3}{4}[\frac{r}{r-c}]$ and price $P^* = \frac{c+2r}{3}$, split the market equally, and earn symmetric levels of profits $\pi^* = \frac{1}{6}[r - c]$. This corresponds to an adjacent-markets equilibrium outcome in the second stage pricing game.

Proposition 4 can be viewed as the limiting case of technological progress reducing the cost of increasing platform scope. For brevity, we focus our discussion on the symmetric equilibrium. It is somewhat striking that the equilibrium choices of scope (and the value provided by products) is limited even when scope is costless. In other words, not only is it strategically sensible to limit product convergence (as discussed following Proposition 2), but this choice is also an equilibrium strategy. The restricted level of convergence that emerges as subgame-perfect in Proposition 4 actually reflects the *optimal balance* between the value effect and the substitution effect that we have highlighted as characterizing digital convergence. Figure 4.3(a) shows that equilibrium duopoly prices are higher than they would be under a monopoly market structure at the same level of product scope; moreover, equilibrium profits are the highest possible under any symmetric duopoly equilibrium. The symmetric levels of convergence are therefore Pareto-efficient as well.

For industries in which firms' product scope choices are not entirely driven by industry standards or exogenous technology factors, an important strategic implication is therefore that after a point, these choices should be determined *purely* by strategic considerations. Managers should resist being influenced by the technological possibilities of designing highly versatile products that consumers might value, and should limit the extent to which their products can fulfill customer needs even if more extensive fulfillment is both technologically viable and cost effective. Strategic considerations of this kind may have limited the extent of convergence in the past between the cable and telecommunication industries, thereby enabling firms in both these industries to maintain relatively high prices for their services that bundle broadband with video or telephony. However, the recent entry of cable companies into the local telephony industry, and a corresponding move by companies like Verizon into cable television delivered over fiber-optic lines suggests that they are moving away from their strategically sensible choices and towards ones driven by technological feasibility.

An immediate corollary of Proposition 4 is that while technological investments that lower variable costs c lead to lower prices, firms retain a substantial fraction of their cost reductions. Further, this effect on price is entirely indirect, through the equilibrium change in their choice of product scope $\frac{1}{t^*}$. Recall from Section 4 that under an adjacent-markets equilibrium with fixed product scope, the duopolists did not reduce prices when c was reduced[15].

[15] Moreover, the equilibrium level of scope chosen by the duopolists is lower than the socially efficient level. At the socially efficient level of scope, each product satisfies every functionality requirement of the consumer perfectly ($t = 0$), thus resulting in the highest amount of surplus possible. However, the products are also perfect substitutes, leading to undifferentiated Bertrand competition and marginal cost pricing. Interestingly, a monopolist would find it optimal to provide an infinite level of product scope, thereby achieving the first best outcome. However, the monopolist would also

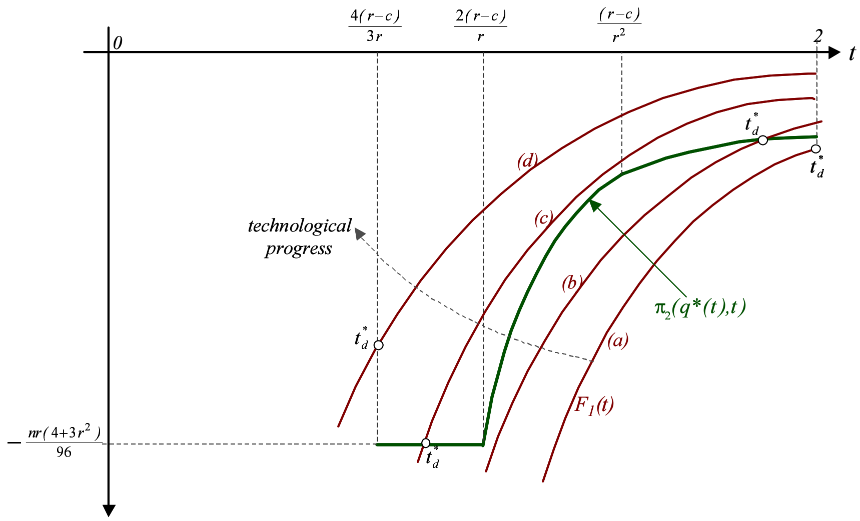

Figure 5.1: Equilibrium choice of product scope for different marginal costs of scope. $\pi_2(q^*(t),t)$ is marginal gross profit and the four $F_1(t)$ curves represent four different levels for marginal cost of scope.

When fixed costs are non-zero and increasing scope is costly, firms may find it profitable to choose a level of scope lower than the equilibrium value derived in Proposition 4. If there is a unique point t_d^* that satisfies conditions (5.1-5.3), this is the unique symmetric equilibrium. The curves *(b)* and *(c)* in Figure 5.1 illustrate two candidate marginal fixed costs curves, and their first-stage equilibrium levels of scope, which lead to second-stage local-monopoly and adjacent-markets equilibria respectively.

If there are no feasible values of product scope that satisfy the analytical equations (5.1-5.3), one needs to examine the value of $\Pi_1^i(t,t)$ at the boundary points $t = \frac{4[r-c]}{3r}$ and $t = 2$. Curve *(d)* in Figure 5.1 illustrates one such outcome where $F_1(t)$ lies entirely above the second-stage equilibrium marginal profit curve. Product scope at the unique symmetric subgame perfect equilibrium in this case is identical to that obtained with costless scope, and strengthens Proposition 4 to the extent that its result continues to hold if the marginal cost of scope is non-zero but small. Curve (a) in Figure 5.1 illustrates the case where $F_1(t)$ lies entirely below the second-stage equilibrium marginal profit curve. The symmetric subgame perfect equilibrium in this case corresponds to $t_d^* > 2$, and therefore, we do not interpret this further[16].

end up appropriating the entire value created, leaving no surplus to the consumers.

[16] If a unique value for t_d^* exists, that satisfies (5.1)and (5.2) but not (5.3), then it is a local minimum. In this case, either $t_d^* = \frac{4}{3}(\frac{r-c}{r})$ or $t_d^* > 2$ could be the equilibrium. Very little can be said for the case where multiple points satisfy

6. Non-exclusive and incremental purchases

In this section we present additional results that are obtained when we relax our assumptions on the maximum breadth of functionalities demanded and on the minimum level of product scope. When $r > \frac{1}{2}$, we show that a fourth kind of equilibrium configuration – the non-exclusive equilibrium – also exists in the duopoly model[17], for that range of parameter values under which a competitive equilibrium configuration was previously the unique outcome, and we discuss some of its properties. We also illustrate how when platform scope is low ($t > 2$) and r is close to 1, outcomes in which consumers purchase both products can occur even when the equilibrium configuration is local monopoly. Our primary objective in providing these illustrations is to highlight some unique outcomes and their corresponding purchasing patterns that digital convergence may cause.

6.1. Non-exclusive equilibrium: when consumers purchase multiple converged products.

When $r > \frac{1}{2}$, the exact analytical expressions for consumer utility, while still based on (3.1), are slightly different from those in (3.2)[18]. However, when prices are high enough, the inverse demand curves are structurally similar to those described in Section 4, and the first three equilibrium configurations – local monopoly, adjacent-markets, and competitive – continue to be feasible. With a symmetric, exogenously specified level of product scope, exactly one of these three configurations occurs in each segment of the permissible range of t values. Though the ranges themselves are different, the basic structure derived in Section 4 is preserved – as t decreases, the equilibrium transitions from local monopoly to adjacent-markets, and then to competitive. Table 6.1 summarizes the relevant ranges of t, and the corresponding outcomes. At $r = \frac{1}{2}$, the results coincide with the corresponding results derived in Section 4 and therefore there is no discontinuity.

More importantly, the fourth kind of equilibrium configuration described in Section 4.2 is now feasible. It arises when prices fall to the point where, rather than inducing consumers to switch

(5.1 – 5.3) and the equilibrium in this case depends on the specific form of the fixed cost function.

[17] Most of the monopoly results obtained for $r < \frac{1}{2}$ are preserved here; we present details in the extended appendix.

[18] These exact expressions are available in the extended appendix.

Range of values of t	**Equilibrium configuration**	**Demand per firm**	**Equilibrium prices**
$4 \geq t \geq \frac{4[r-c]}{4r^2-6r+3}$	Local monopoly	$nq^*(t) = n\sqrt{\frac{r-c-\frac{r^2 t}{4}}{3t}}$	$P^*(t) = \frac{2r+c}{3} - \frac{r^2 t}{6}$
$\frac{4[r-c]}{4r^2-6r+3} \geq t \geq 4[r-c]$	Local monopoly	$nq^*(t) = \frac{n[4[r-c]-t[2r-1]]}{8t[1-r]}$	$P^*(t) = \frac{4[r+c]-t[2r-1]}{8}$
$4[r-c] \geq t \geq \frac{4[r-c]}{2-r}$	Adjacent-markets	$nq^*(t) = \frac{n}{4}$	$P^*(t) = r[1-\frac{t}{4}]$
$\frac{4[r-c]}{2-r} \geq t \geq 0$	Competitive	$nq^*(t) = \frac{n}{4}$	$P^*(t) = c + \frac{t[1-r]}{2}$
$\frac{4[r-c]}{2-r} \geq t \geq \frac{128}{9}c$	Non-exclusive (for high r)	see Table 6.3	see Table 6.3

Table 6.1: Equilibrium configurations when $r > 1/2$)

Range of values of r	**Incremental value function $I(y,t)$**	
$\frac{1}{2} \leq r \leq \frac{3}{4}$	$I(y,t) = \frac{t}{8}$	for $\frac{1}{2} \leq y \leq \frac{1+2r}{4}$
	$I(y,t) = t[\frac{1}{8} - t[y - \frac{1+2r}{4}]^2]$	for $\frac{1+2r}{4} \leq y \leq \frac{1+r}{2}$
	$I(y,t) = t[\frac{3+2r}{4} - y]^2$	for $\frac{1+r}{2} \leq y \leq \frac{5-2r}{4}$
	$I(y,t) = t[[\frac{3+2r}{4} - y]^2 + [y - \frac{5-2r}{4}]^2]$	for $\frac{5-2r}{4} \leq y \leq 1$
$\frac{3}{4} \leq r \leq 1$	$I(y,t) = \frac{t}{8}$	for $\frac{1}{2} \leq y \leq \frac{1+2r}{4}$
	$I(y,t) = t[\frac{1}{8} - t[y - \frac{1+2r}{4}]^2]$	for $\frac{1+2r}{4} \leq y \leq \frac{5-2r}{4}$
	$I(y,t) = t[\frac{3+2r}{4} - y]^2$	for $\frac{5-2r}{4} \leq y \leq \frac{1+r}{2}$
	$I(y,t) = t[[\frac{3+2r}{4} - y]^2 + [y - \frac{5-2r}{4}]^2]$	for $\frac{1+r}{4} \leq y \leq 1$

Table 6.2: The incremental value function)

products, incremental demand is derived by inducing them to buy an *additional* product. As a consequence, the inverse demand function of firm i is determined not by the difference in total value between the two products, but by the *incremental value* that product i provides to a consumer who already owns product j.

Under symmetric scope across products, let $I(y,t)$ be the *incremental value function*, which specifies the additional value that the consumer located at a distance $q \in [0, \frac{1}{2}]$ from product i obtains from product i if she already owns product j. The function is defined recursively as:

$$I(q + \frac{1}{2}, t) = \max\{0, U(q + \frac{1}{2}, t) - U(1-q, t) + I(1-q, t)\},$$

and is economically relevant only when neither product dominates the other on all of a consumer's desired functionalities[19].Figure 6.1 illustrates the function $I(y,t)$.

[19] If product j provides a higher quality than product i on the entire range of functionalities that a consumer cares about, then this incremental value is simply zero. If on the other hand, product i is superior to product j on the entire relevant range of functionalities, the incremental value is simply the difference between the gross values provided by the two products. In either of these cases, under symmetric prices, consumers have no incentive to buy both products.

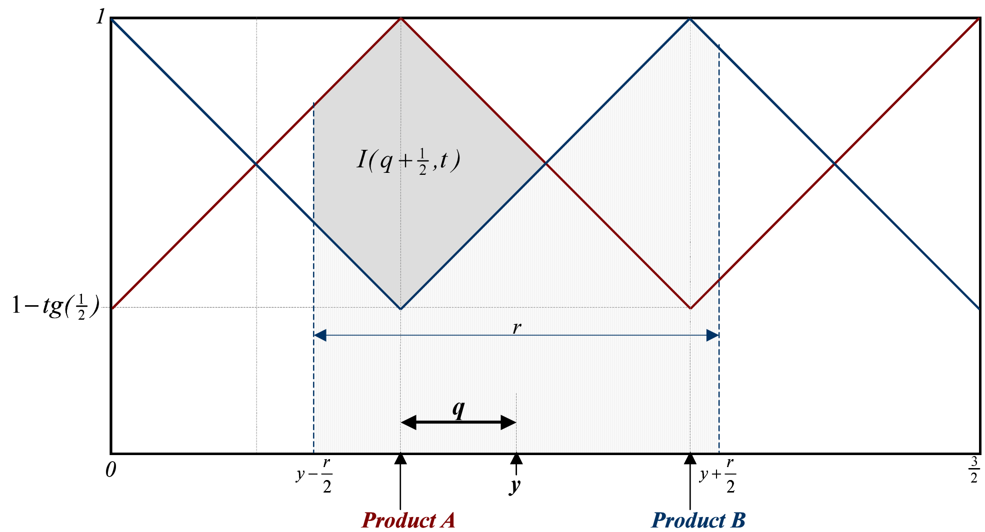

Figure 6.1: Illustrates the incremental value function for a high value of r.

Table 6.2 summarizes the algebraic expressions of $I(y,t)$. For $r \geq \frac{1}{2}$, $I(y,t)$ is monotonically decreasing in y, but positive everywhere. Under symmetric scope, the consumer who has the highest incremental value from a second product will be located halfway between the two product locations i.e. at $y = \frac{3}{4}$. This consumer will be indifferent between the two products when their prices are equal. At sufficiently low prices, i.e. if $p_j \leq I(\frac{3}{4},t)$, the inverse demand function changes slope discontinuously from its value in the competitive region and a third region, which we term the non-exclusive region, emerges. The demand function in this non-exclusive region is more elastic than in the corresponding competitive region, and actually has the same slope as the monopoly inverse demand function. In summary, for $p_j \leq I(\frac{3}{4},t)$:

$$\begin{aligned} P_i(q_i,t,t,p_j) &= U(q_i+\frac{1}{2},t) - U(1-q_i,t) + p_j \quad \text{for} \;\; 0 \leq q_i \leq q_{NE}; \qquad (6.1) \\ &= I(q_i+\frac{1}{2},t_i,t_j) \quad \text{for} \;\; q_{NE} \leq q_i \leq \frac{1}{2}, \end{aligned}$$

where q_{NE} is defined using the equation $I(1-q_{NE},t) = p_j$.

Range of values of r	**Equilibrium demand per firm**	**Equilibrium prices**
$\frac{3-\sqrt{3}}{2} \leq r \leq \frac{9-\sqrt{5}}{8}$	$nq^*(t) = n[\frac{2r-1}{6} + \frac{\sqrt{4r^2-4r+7}}{12}]$	$P^*(t) = \frac{t}{8} - \frac{[\sqrt{t[1-2r]^2}+\sqrt{[7-4r[1-r]]t}]^2}{144}$
$\frac{9-\sqrt{5}}{8} < r \leq \frac{7}{8}$	$nq^*(t) = \frac{n[5-4r]}{32[1-r]}$	$P^*(t) = \frac{[5-4r]t}{16}$
$\frac{7}{8} < r \leq 1$	$nq^*(t) = \frac{n}{2}$	$P^*(t) = \frac{t1-2r]^2}{8}$

Table 6.3: Outcomes under the non-exclusive equilibrium)

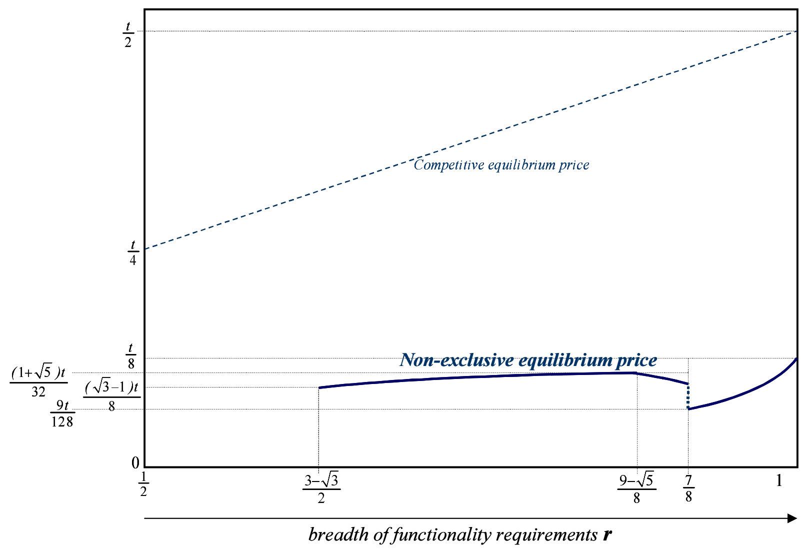

Figure 6.2: Illustrates how the non-exclusive equilibrium price varies as the breadth of functionality requirements increases, and its value relative to the competitive equilibrium price.

For a high enough value of r, a symmetric non-exclusive equilibrium always exists. Table 6.3 summarizes the equilibrium outcomes for different values of r, when marginal costs are zero. Figure 6.2 depicts the equilibrium prices $P^*(t)$ as a function of r. Given the diametrically opposite location of firms at $\frac{1}{2}$ and 1, an equilibrium demand of $nq > \frac{n}{4}$ implies that the markets start overlapping in the middle, and that consumers with locations $y \in [1-q, \frac{1}{2}+q]$ buy *both products*.

Of particular interest is the case of $r \geq \frac{7}{8}$, where every consumer in the market buys both products, and prices drop to the incremental value of the consumer who values the product the least. This price is far lower than the actual value that any consumer gets from the product she buys, and is also lower than the corresponding competitive equilibrium price at the same level of scope. As a result, the consumer surplus will be much higher under the non-exclusive equilibrium. Interestingly, total surplus is also strictly higher than under the competitive equilibrium configuration at the same level of scope, because a fraction (sometimes all) of consumers buy both products, and use the *more efficient* of the two products to satisfy each functionality requirements.

6.2. Broad functionality requirements and very low platform scope

Finally, we discuss a market in which products have very low scope. This represents a scenario prior to digital convergence, where there is absolutely no overlap in the functionalities provided by the two products. Each firm is a monopolist, and prices accordingly. However, at very high values of r, both products may provide some consumers with positive value, although on distinct subsets of their desired functionalities.

We consider the specific case where scope is symmetric, $t > 4$ and $1 - \frac{1}{t} \leq r \leq 1$. Under these constraints, the consumer's value function is[20]:

$$U(y,t) = \begin{cases} \frac{1}{t} & \text{for } \frac{1}{2} \leq y \leq \frac{1+r}{2} - \frac{1}{t}; \\ \frac{1}{2t} - [y - \frac{1+r}{2}][1 + \frac{t}{2}[y - \frac{1+r}{2}]] & \text{for } \frac{1+r}{2} - \frac{1}{t} \leq y \leq \frac{3-r}{2} - \frac{1}{t} \\ \frac{1}{t} - [1-r][1 + t[1-y]] & \text{for } \frac{3-r}{2} - \frac{1}{t} \leq y \leq \frac{1+r}{2} \\ \frac{1}{t} - [1-r] + t[[1-y]^2 + [\frac{1-r}{2}]^2] & \text{for } \frac{1+r}{2} \leq y \leq 1 \end{cases} \tag{6.2}$$

The profit function is similar to the monopoly gross profit function – there is at most one interior maximum, and profits are maximized either at this interior point, or at full market coverage. For $c = 0$, the optimal prices for each firm are summarized in Table 6.4. Even though the firms are local monopolists and do not influence each others prices or profits, it is clear that there is some overlap in the consumers served by both products[21]. As the span of consumers' functionality requirements increases, a larger fraction of the consumers buy both products. For values of r sufficiently close to 1, all consumers in the market buy both products.

This example is particularly interesting when contrasted with the other non-exclusive equilibrium

[20] Interestingly, with non-overlapping functionalities provided by the two products, a product's value function is identical to the incremental value function that was used in deriving the non-exclusive equilibrium in the last subsection.

[21] At the margin, with $t = 4$ and $r = 1 - \frac{1}{4} = \frac{3}{4}$, $q^*(t) \simeq 0.29 > \frac{1}{4}$.

Range of values of r	Equilibrium demand per firm	Equilibrium prices
$1-\frac{1}{t} \leq r \leq \frac{5t-2-\sqrt{t^2-4t+20}}{4t}$	$nq^*(t) = \frac{n[2rt-4+\sqrt{rt[rt-4]-28}]}{6t}$	$P^*(t) = \frac{24-[rt-2]^2+[rt-2]\sqrt{rt[rt-4]+28}}{36t}$
$\frac{5t-2-\sqrt{t^2-4t+20}}{4t} \leq r \leq \frac{t^2-2}{t^2}$	$nq^*(t) = \frac{2+t[1-r][t-2]}{4t^2[1-r]}$	$P^*(t) = \frac{2+t[1-r][t-2]}{4t}$
$\frac{t^2-2}{t^2} \leq r \leq 1$	$nq^*(t) = \frac{n}{2}$	$P^*(t) = \frac{[2-t[1-r]]^2}{4t}$

Table 6.4: Equilibrium demand and prices for very low levels of scope)

described in the last subsection. It suggests that as digital technology progresses and industries converge, consumers who started out buying multiple specialized products first switch to a single general-purpose product, and then later may begin buying multiple products again, using the additional general-purpose products as if they were specialized.

7. Discussion

This paper has presented a model of competition between platform-based products in converging information technology markets. The model provides a careful and rich representation of how products fulfil diverse consumer functionality requirements, how the effectiveness of these products varies with scope, and how this affects value and product differentiation when industries converge. It represents a new contribution to the economics of information technology, generalizing standard horizontal differentiation and product characteristics models of imperfect competition, explicitly separating product design choices from consumers' preferences, characterizing four different kinds of pricing equilibrium outcomes, and enabling the derivation of equilibria involving multiple purchases. A number of our results attest to the usefulness of these modeling enhancements.

Our key results and their implications are summarized below:

– At early and intermediate stages of digital convergence, the value effect dominates the substitution effect. Therefore, despite a growing overlap in functionality between converging technology products, firms should expect a bilateral increase in price and profits. However, at a later stage of convergence, the substitution effect dominates the value effect, bilaterally reducing prices and profits. When convergence is driven by exogenous factors (technological progress in an upstream/downstream industry, shared technological standards, intra-industry competition), managers must therefore be careful not to overestimate the benefits or sustainability of the early gains their companies may realize from digital convergence.

– When the extent of digital convergence can be endogenously controlled by firms, it is crucial that product managers place strategic considerations ahead of technological possibilities. It is only through careful and appropriate strategic control of the scope of one's product that a firm can sustain the higher revenues that convergence makes feasible. The importance of such control was emphasized in a recent interview by Steve Jobs, who noted that "the key to succeeding in the converged economy is resisting the temptations to enter certain markets and to know when to say no. I'm as proud of what we don't do as what we do" (Baker and Green 2004). Our results show that a restricted choice of scope under which both prices and profits are bilaterally maximum across both converging industries is supported as a subgame-perfect Nash equilibrium, even when increasing the scope of one's products is costless.

– Managerial incentives to invest in technology that reduces their variable costs of production (for instance, by shifting device logic from hardware to software) are maximum at an intermediate stage of digital convergence; a firm can capture a significant fraction of the reduction in variable cost their investment realizes. In contrast, at an advanced stage of convergence, any cost reduction from these investments will be competed away, captured by customers via lowered prices.

– Industry-wide efforts that result in an increase in the span of functionality that each customer values are likely to be more profitable over time than those which increase the scope of products. While both of these changes increase both the value of products as well as their substitutability, the former always enable firms to increase prices and profits, independent of the extent of digital convergence.

Our analysis also highlights some interesting issues related to efficiency and market structure. We find that prices in competitive converging industries are sometimes higher than their monopoly counterparts. This counter-intuitive result is partially a consequence of the higher product variety (and hence, a higher average product value) in the competitive market as compared to a single-product monopoly. Indeed a monopolist supplying two products instead of one would actually charge prices no less than the equilibrium duopoly prices, but depending on the fixed product development costs, it may or may not be optimal for a monopolist to offer multiple products. As discussed earlier, we also find that competitive firms in converging industries might significantly restrict their level of product scope, even when scope is costless. In this scenario, convergence can reduce total surplus, since a monopolist would provide a much higher level of product scope when faced with the same demand and cost of scope; however, the monopolist would also appropriate most or all of this higher level of surplus, leaving very little for the consumers.

Finally, our analysis of non-exclusive and incremental purchases suggests an interesting trajectory in purchasing patterns that may accompany sustained digital convergence. When products are relatively specialized, consumers are forced to purchase multiple products to satisfy their requirements, as no single product is sufficiently effective over the entire set of functionalities they desire. As product scope increases, each product becomes significantly more effective on a broader range of functionalities and consequently consumers shift to buying fewer products. If subsequent convergence is extensive, consumers may once again shift to buying multiple products; the latter shift is caused by the fact that at very high levels of convergence, prices drop to the extent that purchases can be justified based solely on the incremental value each product provides. While each product is very effective at satisfying a consumer's entire span of desired functionalities, consumers will therefore still purchase multiple products and use these general-purpose products as if they were specialized; multi-computer households that buy both PCs and Macs are a good example of this phenomenon.

This trajectory of purchasing patterns may in fact be cyclical. The emergence of new categories of functionalities, especially driven by disruptive technological innovations (Nault and Vandenbosch, 2000), or gradual changes in quality expectations by consumers may alter the space of functionalities and the partial-equilibrium utility functions of consumers. This could result in firms' broad product scope choices as eventually being perceived as specialized again, and a repetition of this cycle when the next generation of technology emerges. Microsoft's attempts to take personal computing technology into the living room with the XBox platform and Windows Media Center might indicate the start of a new cycle of convergence in consumer electronics. As documented by Mendelson and Pillai (1998), product lifecycles in the IT industry are progressively shorter over time, and these cycles are likely to become increasingly rapid. Incorporating this kind of cyclical effect into a formal model is an interesting direction for future research.

A limitation of this paper is that it does not explicitly model intra-industry competition. Our analysis in section 4 sheds some light on this, though indirectly, and suggests that if there are intra-industry competitive factors that dictate product scope choices, converging markets will intensify competition and prices will fall as product scope increases. In contrast, using the flexibility afforded by digital convergence to expand into a different industry may be a strategic response to increased competition within one's own industry (this may explain, for instance, why Handspring initially chose to incorporate voice communication into their product via the Springboard). A more precise analysis of differentiated competition within each industry with the simultaneous threat of convergence from a neighboring industry remains another promising line of future work.

8. References

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9. Appendix: Outline of proofs

This appendix presents an outline of the proofs. Details are available in the extended appendix.

Lemma 1: (a) The continuity of $U(y,t)$ is established by verifying its values from below and from above at $y = \frac{1+r}{2}$ and $y = \frac{2-r}{2}$, the two points at which its functional form changes. Computing the expressions for $U_1(y,t)$ and $U_2(y,t)$ verify that both are negative.

(b) The continuity of $U_1(y,t)$ is established by verifying its values from below and from above at $y = \frac{1+r}{2}$ and $y = \frac{2-r}{2}$, and inspection of its expression establishes that it it piece-wise differentiable.

(c) Computing the expression for $U_{12}(y,t)$ verifies that it is negative.

Lemma 2: The proof defines the following functions:

$$R^M(q,t) = nq[P^M(q,t) - c]. \tag{9.1}$$

$$R^C(q_i,t_i,t_j,p_j) = nq_i[P^C(q_i,t_i,t_j,p_j) - c]. \tag{9.2}$$

The function $\pi^i(q_i,t_i,t_j,p_j)$, $i = A,B$ takes either the form $R^M(q_i,t_i)$ or the form $R^C(q_i,t_i,t_j,p_j)$. The former is the monopoly profit function, which is shown in Lemma 4 to have at most one interior maximum in $[0,\frac{1}{2}]$. The function $R^C(q_i,t_i,t_j,p_j)$ is shown to be strictly concave for $q_i \leq \frac{1-r}{2}$ and to have no more than one interior maximum in $[0,\frac{1}{2}]$. Finally, the condition $R_1^M(q^*,t) = 0$ for any q^* is shown to imply that $R^C(q,t_i,t_j,p_j) < 0$ for $q > q^*$, which implies that both $R^M(q,t_i)$ and $R^C(q,t_i,t_j,p_j)$ cannot simultaneously have interior maxima.

Proposition 1: This is a fairly elaborate proof. Broadly, for any (t_A,t_B), the following two functions are defined:

$$\begin{aligned} Q^M(t_i) &= x : \frac{U(x+\frac{1}{2},t_i) - c}{-U_1(x+\frac{1}{2},t_i)} = x, \text{ if such an } x \text{ exists in } [0,\frac{1}{2}] \\ &= \frac{1}{2} \text{ otherwise.} \end{aligned} \tag{9.3}$$

$$\begin{aligned} Q^C(t_i,t_j) &= x : \frac{U(x+\frac{1}{2},t_i) - c}{-[U_1(x+\frac{1}{2},t_i) + U_1(1-x,t_j)]} = x, \text{ if such an } x \text{ exists in } [0,\frac{1}{2}] \\ &= \frac{1}{2} \text{ otherwise.} \end{aligned} \tag{9.4}$$

$Q^M(t_i)$ is the interior maximum in q (if it exists) of the *monopoly* portion of the profit function $\pi^i(q, t_i, t_j, p_j)$, and is $\frac{1}{2}$ otherwise. Analogously, $Q^C(t_i, t_j)$ is the interior local maximum of the *competitive* portion of the function $\pi^i(q, t_i, t_j, p_j)$ with $p_j = U(1 - Q^C(t_i, t_j), t_j)$. The proof of the proposition then shows the following relationship between these functions and the existence of the different kinds of equilibria:

- A local-monopoly equilibrium is feasible only if $Q^M(t_A) + Q^M(t_B) < \frac{1}{2}$
- An adjacent-markets equilibrium is feasible only if $Q^M(t_A) + Q^M(t_B) \geq \frac{1}{2}$ and $Q^C(t_A, t_B) + Q^C(t_B, t_A) \leq \frac{1}{2}$
- A competitive equilibrium is feasible only if $Q^C(t_A, t_B) + Q^C(t_B, t_A) \geq \frac{1}{2}$.

By definition, $Q^M(t_A) + Q^M(t_B) < Q^C(t_A, t_B) + Q^C(t_B, t_A)$, and this relationship therefore establishes that there is a maximum of one kind of equilibrium configuration for each (t_A, t_B).

The regions in the (t_A, t_B) space over which each of the configurations exist are established by (i) computing the explicit algebraic form of $Q^M(t_i)$ and $Q^C(t_i, t_j)$ for different ranges of (t_i, t_j) and (ii) computing the ranges of (t_i, t_j) over which each of the three conditions above are satisfied. The Nash equilibrium price pairs for each configuration are derived by computing the equilibrium q_A, q_B pairs, and are summarized in Table A.4 in the extended appendix.

Proposition 2: This follows directly from the proof of Proposition 1. When $t_A = t_B = t$, the expression for $Q^C(t, t)$ is considerably simplified, and applying the three conditions above yield the ranges of scope values in column 1 of Table 4.1. The equilibrium price pairs for each equilibrium have been computed in the proof of Proposition 1, and substituting $t_A = t_B = t$ yields the expressions in column 3 of Table 4.1. Once these prices and demand values are known, computing the payoffs is straightforward.

Lemma 3: (a) For a local-monopoly equilibrium, the result obtains from a direct application of the envelope theorem. For an adjacent markets equilibrium, the proof examines the changes in payoffs for a small decrease in scope, represented by a change in t_i to $t_i + \varepsilon$. Since there are multiple equilibria for most (t_A, t_B) pairs, it is assumed that if the choice of quantities prior to the increase of ε continues to be an equilibrium, the firms remain at this equilibrium, and the result follows from the fact that $R_2^C(q_i, t_i, t_j, p_j) < 0$. If not, the firms move to a new quantity pair that was not an equilibrium before the increase of ε; the proof shows that this must always lead to a decrease in the equilibrium demand for firm i, and consequently, payoffs decrease.

(b) An algebraic expression for the equilibrium profit function is computed in the proof, and its total derivative with respect to t_i is shown to be strictly positive.

Proposition 3: (a) These are simply first-order necessary and second-order sufficient conditions on the payoff functions for $(1/t_d^*)$ to be a first-stage Nash equilibrium.

(b) Lemma 3(b) shows that under any competitive equilibrium, a unilateral decrease in scope by firm i (that is, and increase in t_i) increases revenues; at best, this decrease in scope leaves costs unchanged (or reduces them). Therefore, firm i can increase profits by increasing t_i. As a consequence, the second-stage equilibrium has to be either local-monopoly or adjacent markets.

Proposition 4: (a) Lemma 3(a) shows that for any first-stage candidate pair (t_A, t_B), which corresponds to a local-monopoly or adjacent-markets equilibrium subgame, firms have a unilateral incentive to increase their scope if it leaves them in the same equilibrium configuration. Further, along the AM locus the payoff functions are continuous and decreasing in t_i. Therefore, these cannot be part of any subgame perfect equilibrium. Along the CA locus, an increase in t_i takes the firm into the adjacent-markets region while a decrease takes it into the competitive region. Both those changes strictly reduce payoffs. As a consequence, any pair (t_A, t_B) along the CA locus for which a pure-strategy second stage equilibrium exists is part of a subgame perfect equilibrium.

(b) Given (a), the only feasible symmetric subgame perfect Nash equilibrium is the one under consideration. For symmetric values of scope, a pure strategy second-stage price equilibrium always exists. The proof simply computes the appropriate algebraic expressions for equilibrium price and payoffs.