Strategic Experimentation in Networks

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Abstract: This paper examines experimentation and innovation when experimental results diffuse along social links. This public-goods nature of experimentation is a feature of many areas of economics, such as consumer choice, research and development, and agriculture. The paper asks: How do different patterns of social and geographic links affect experimentation? Who experiments and who free rides? Do more links enhance or diminish social welfare? The analysis finds, first, that social networks can foster specialization. In every social network there is an equilibrium where some individuals experiment and others completely free-ride. In many networks this extreme is the only equilibrium outcome. Second, specialization can benefit society as a whole. This outcome arises when specialists are linked, collectively, to many agents. Finally, new links can reduce overall welfare. A new link increases access to new results, but also reduces an individual's incentives to conduct his own experiments. Hence, overall welfare can be higher when there are holes in a network.

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I. Introduction

This paper builds a network model of strategic experimentation. Individuals experiment and the results are non-excludable along social links. We see this public goods nature of experimentation in many areas of economics. Consumers benefit from research of friends and family [e.g., Feick and Price (1987)]. In medicine and other technical fields, professional networks can shape research patterns [e.g., Coleman et al. (1957), Burt (1987), Valente (1995)]. In agriculture, one farmer's experience with a new crop can benefit other farmers, and the physical and social geography of the countryside can influence experimentation and learning [e.g., Foster and Rosenzweig (1995), Conley and Udry (2002), Munshi (2003)]. In industry, it has been long posited that research findings spillover to other firms. Information spread is often local and thus can depend on social networks and the geography of industry and trade [e.g., Jaffe et al. (1993), Breschi & Lissoni (2003), Guiso & Schivardi (2004)]. In all these settings, social structures and geography can influence the incentive to innovate.

These findings raise a new set of research questions. How does the social or geographic structure affect the level and pattern of experimentation? Do people experiment themselves or rely on others? How do new links - links between communities or firms, for example - affect experimentation and welfare?

This paper builds a model to address these questions. There is fixed social structure. People desire new information and experimentation is costly. The results of experiments diffuse to linked agents. Individuals decide how much to experiment knowing their results diffuse to their social or geographic neighbors.

Our analysis yields three main insights. First, social networks can foster specialization. In every social network there is an equilibrium where some individuals experiment and others completely rely on their efforts. In many, particularly asymmetric, social networks this extreme form of free-riding is the only equilibrium outcome. In all social networks, such patterns are the only stable outcomes. Hence, an agent's position in a network can determine whether or not they contribute to research and experimentation. Second, this specialization can have welfare benefits. It may benefit the society as a whole for some agents to specialize in experimentation and others to rely on their results. This outcome arises when specialists are linked, collectively, to many people in society. Finally, new links can reduce overall welfare. A new link increases access to information, but also reduces an individual's incentives to conduct his own research. Hence, overall welfare can be higher when there are holes in a network – not everyone should be linked.

The literature on strategic experimentation has neglected network aspects, so far. Foster & Rosenzweig (1995) build a model of strategic experimentation and test the model using data from villages in India where farmers experimented with new high-yield-variety (HYV) crops. They find evidence that results of experiments are a public good and that people free-ride; the level of experimentation is less than is socially optimal. Bolton and Harris (1999) set out a general theory; they consider a two-armed bandit problem when a group of individuals can all observe the outcome of all others' experiments. The results show that, while current experimentation discourages current experimentation by others, current experimentation can encourage future experimentation as players learn more about the underlying distribution of the payoffs. Our innovation is the social network. We build a very simple model of experimentation and explore how the link structure affects outcomes. We find that network characteristics have first-order effects on experimentation patterns.

This paper has two parts. In the first, we study general networks. We characterize equilibrium effort patterns, their stability, their welfare properties, and the effect of new communication links on experimentation and welfare. In the second part, we construct specific network models to capture main ideas in sociological and other literatures concerning patterns of experimentation and communication. Our *Overlapping Neighborhoods* model captures a symmetric structure where people learn from those close in geographic or social space, e.g., those who live in the same neighborhood or have the same ethnicity. We explore increasing levels of societal integration. Our *Communities/Bridges* model captures an asymmetric structure where people are divided into disjoint communities, such as villages or firms. We explore increasing numbers of links – bridges – between communities. Our *Core-Periphery* model captures an asymmetric, hierarchical structure. We explore increasing density of links between core and periphery.

With these models, we refine our general insights to specific settings. In our study of overlapping neighborhoods, for example, we find that specialization has greater benefits when society is more integrated. Local specialists can reach more people as the size of these neighborhoods expand. In our study of communities, we find that new bridges have mixed effects. In the sociology literature, bridges are beneficial because they bring new information into a community [e.g., Granovetter (1973)]. We find that when an agent gains access to an outside source of information, he may reduce his own research effort. This reduction has negative externalities. Welfare may not fall, but there are distributional consequences. In our study of core-periphery structures, we find that agents in the core can experiment relatively less than agents in the periphery. Core agents have greater access to other agents' information and hence have less incentive to expend their own effort. This result corresponds closely to the argument that innovation often occurs in the peripheries of social systems (e.g., Kuhn (1963), Jewkes et al. (1959), and Hamberg (1963)).

This paper complements a growing interest in networks and social learning. Empirical studies now consider whether people gain information from those to whom they are socially linked, and a new set of papers attempts to identify network effects on technology adoption (Bandiera and Rasul (2001), Conley and Udry (2002), and Munshi (2003)). In the theory of social learning, a few papers consider social networks: Bala & Goyal (1998, 2001) and Cowan & Jonard (2004) construct models where agents learn the choices and payoffs only of linked individuals; they do not address the initial generation of new information.[1] We show that this generation depends much on the network.

This paper also contributes to the theoretical literature on networks in economics.[2] It provides the first analysis of a game played on a network when actions are strategic substitutes. We especially introduce a new notion from graph theory to characterize the Nash equilibria (see Proposition 3). This notion provides a natural, geometric expression of the interplay between strategic substitutability and the network structure. The paper further develops a new research strategy for the study of networks. It is often difficult, if not impossible, to obtain analytical results for general graphs. Moreover, real world networks typically exhibit structural regularities. We address both issues by constructing families of graphs to represent different social environments. There is enough structure to obtain analytical results, and we model a precise en-

[1] For review of the social learning literature see Bikhchandani et al. (1998) and Cao and Hirshleifer (2000).

[2] Much work on networks considers the general properties of equilibrium networks. Contributions include Myerson (1977), Aumann and Myerson (1988), Jackson and Wolinsky (1996), Bala and Goyal (2000), Jackson and Watts (2002). For a compresive collection see Dutta and Jackson (2003) and their (2003) review. There is a growing literature which builds network models to represent different economic settings. Young (1999) and Morris (2000) examine coordination games played in social networks. Bramoullé (2002a, 2002b) examines anti-coordination games. Boorman (1975), Tassier (2000), Calvó-Armengol (2004), and Calvó-Armengol and Jackson (2004) study job contact networks. Kranton and Minehart (2001) build a theory of networks of buyers and sellers. Goyal and Moraga (2001) look at R&D collaboration networks between firms. Chwe (2000) studies coordination games and communication networks. Bala and Goyal (1998, 2001) also consider communication graphs. Hendriks et. al. (1999) studies airline networks.

vironment. Our Overlapping Neighborhoods, Communities/Bridges, and Core-Periphery models could be useful in a variety of other applications.

The paper is organized as follows. In the next section, we present the model and in Section III, we study general communication graphs. In Section IV, we develop and analyze our families of graphs. In Section V, we discuss the robustness of our findings to changes in the model's specifications. Section VI concludes.

II. The Model

A. Networks and Strategic Experimentation

In this section, we develop a model of strategic experimentation in a network. There are n agents, and the set of agents is $N = \{1, ..., n\}$. Let e_i denote agent $i's$ level of experimentation, or effort. E.g., e_i could be the amount of land dedicated to a new crop, as in Foster & Rosenzweig (1995). Let $\mathbf{e} = (e_1, ..., e_n)$ denote an effort profile of all agents.

Agents are arranged in a network, which we represent as a graph $\mathbf{g}$, where $g_{ij} = 1$ if agent j benefits directly from the results of agent i's experimentation, and $g_{ij} = 0$ otherwise. We assume that results flow both ways so that $g_{ij} = g_{ji}$. Since agent i knows his own results we set $g_{ii} = 1$. Let N_i denote the set of agents that benefit directly from agent i's experimentation, called i's *neighbors*: $N_i = \{j \in N \backslash i : g_{ij} = 1\}$. Let $k_i \equiv |N_i|$ denote the number of agent i's neighbors. Agent i's *neighborhood* is defined as himself and his set of neighbors; i.e., $i + N_i$.

We make two important assumptions concerning the diffusion of experimental results. First, results diffuse one step and then stop. We make this assumption for simplicity and because it reflects findings that information does not travel more than one or two steps (Friedkin (1983)). The methods we develop here can be extended to diffusion of more than one step (see section V). Second, agents are interested in the same kind of information, and a neighbor's results gives the same benefits as one's own. With these assumptions an agent i derives benefits from the total of his own and his neighbors experimentation: $e_i + \sum_{j \in N_i} e_j$.

We assume each agent receives benefits from experimentation according to a (twice-differentiable) strictly concave benefit function $b(e)$ where $b(0) = 0$, $b' > 0$ and $b'' < 0$. With our assumptions above, an individual i has benefits $b\left(e_i + \sum_{j \in N_i} e_j\right)$. To fix ideas for these payoffs, consider a discrete example: e_i is the number of experimental trials of a new technique. There are many such techniques, each with a different unknown value of output, distributed according to a distribution F. Under the assumption that all trials are independent draws, the expected benefits from the number of trials is the expectation of the first order statistic of e trials. The benefits $b(e)$ are then increasing and concave in e.

We assume the individual marginal cost of experimentation is constant and equal to c. An agent i's payoff from profile $\mathbf{e}$ in graph $\mathbf{g}$ is then

$$U_i(\mathbf{e};\mathbf{g}) = b\left(e_i + \sum_{j \in N_i} e_j\right) - ce_i$$

In section V, we discuss the robustness of our findings to alternative specifications of the model, including convex costs. A main interest of our simple quasi-linear form (besides its analytic tractability) is that it allows us to focus on the effect of the structure.

B. Strategic Interaction

We specify the following game. Given a structure $\mathbf{g}$, agents simultaneously choose experimentation levels. For a profile $\mathbf{e}$, each agent i earns payoffs $U_i(\mathbf{e};\mathbf{g})$. We analyze pure strategy equilibria, as there are no mixed strategy equilibria of this game.[3] In the following analysis, we explore how social structure influences the equilibrium experimentation.

III. Equilibrium Experimentation in General Graphs

A. The Shape of Equilibrium Profiles

We characterize Nash equilibria of the game. Let e^* denote the experimentation level at which, to an individual agent, the marginal benefit is equal to its marginal cost; $b'(e^*) = c$.[4] Let $\overline{e}_i \equiv \sum_{j \in N_i} e_j$ be the total experimentation of i's neighbors. It is easy to see that:

Proposition 1. *A profile* $\mathbf{e}$ *is a Nash equilibrium if and only if for every agent* i *either (1)* $\overline{e}_i \geq e^*$ *and* $e_i = 0$ *or (2)* $\overline{e}_i \leq e$ *and* $e_i = e^* - \overline{e}_i$.

[3] Since the benefit function $b(.)$ is concave and costs are linear, an agent would always earn higher expected payoffs by playing the average of a set of effort levels than a mixture over the set of effort levels. Hence, there is no Nash equilibrium where an agent plays a mixture of effort levels.

[4] Given $b()$ is strictly concave, a level of search $e^* > 0$ exists and is well-defined as long as $b'(0) > c$.

The intuition is straightforward. Agents want to experiment as long as their total benefits are less than $b(e^*)$. Thus, if the benefits they acquire from their neighbors is more than $b(e^*)$, they do no experimentation. If the benefits are less than $b(e^*)$, they experiment up to the point where their benefits are exactly equal to $b(e^*)$.

Proposition 1 tells us that experimentation levels are strategic substitutes. In equilibrium, the more an agent experiments, the less do his neighbors. In general, the equilibrium distribution of experimentation among agents could vary among two extremes. We say a profile $\mathbf{e}$ is *specialized* when every agent either experiments the maximum amount e^* or does no experimentation; for all agents i either $e_i = 0$ or $e_i = e^*$. We call an agent who does the maximum amount, e^*, a *specialist*. We say a profile is $\mathbf{e}$ is *distributed* when every agent does some experimentation; for all agents i, $0 < e_i < e^*$. *Hybrid* equilibria fall between these two extremes.

We illustrate Proposition 1 and these different types of equilibria using three basic graphs, each with four agents. The graphs in Figure 1 - the complete graph, the star, and the circle - represent canonical social structures described in the sociology and geography literature.[5] These graphs are building blocks for the families of graphs we construct later in the paper.

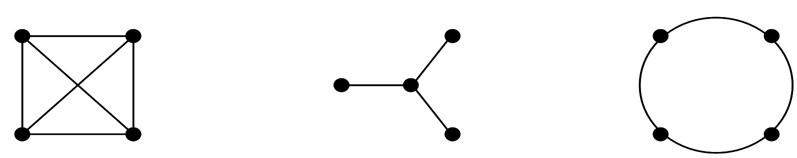

Figure 1. Basic Graphs with Four Agents

Example 1. *Specialized and Distributed Equilibria. Consider the complete graph. In all equilibria there is a total effort of e^* and it can be split in any way among the agents. E.g., effort could be equally distributed, so that each agent experiments $\frac{1}{4}e^*$, or one agent could be a specialist, as shown in panel (a) Figure 2. Note that in all Figures we suppress the term e^* for ease of*

[5] We provide citations and discuss these and other structures below in Section IV. The complete graph represents a densely connected society; all agents know each other. The star graph represents a hierarchical society; only one agent knows everyone else in the society. The circle gives an intermediate case of an equal society, but where each agent only knows a subset of the population.

exposition. On the star, results are quite different. Only specialized profiles are equilibria. Thus the star graph provides a stark example of social structure leading to specialization. There are only two Nash equilibria: Either the center is a specialist, or the three agents at the periphery are specialists, as shown in Figure 2 panel (b). Finally, on the circle, effort can be distributed among the agents, or concentrated among specialists, as shown in Figure 2 panel (c).

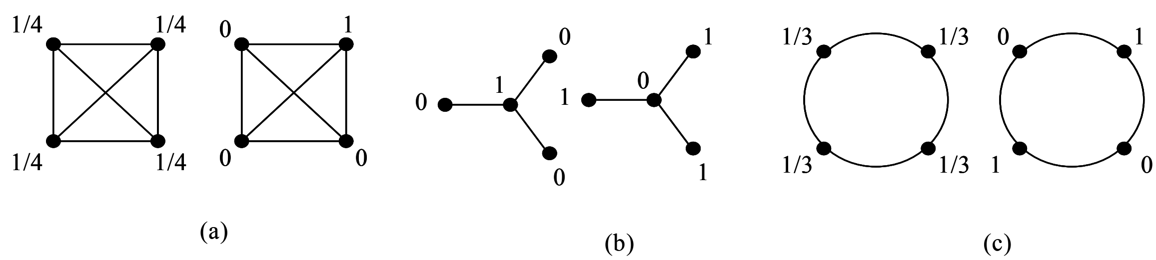

Figure 2. Equilibria in Basic Graphs with Four Agents

The results in Example 1 generalize to complete graphs, stars, and circles for n agents; the Appendix provides the respective corrolaries to Proposition 1.

B. The Existence of Nash Equilibria: Specialized Profiles and Independent Sets

Using standard arguments, it is easy to show that for any social structure there exists a Nash equilibrim profile. Define $f_i(\mathbf{e})$ as the best-response of individual i to a profile $\mathbf{e}$ and define $\mathbf{f}$ as the collection of these individual best-responses $\mathbf{f} = (f_1, ..., f_n)$. From Proposition 1, $f_i(\mathbf{e}) = \max(e^* - \bar{e}_i, 0)$. This implies that $\mathbf{f}$ is a continuous function from the compact convex set $\{\mathbf{e} \in R^n : \forall i, 0 \leq e_i \leq e^*\}$ to itself. By Brouwer's Fixed Point Theorem, $\mathbf{f}$ must have a fixed point. Hence,

Proposition 2. *For any structure* $\mathbf{g}$*, there exists a profile* $\mathbf{e}$ *that constitutes a Nash equilibrium.*

More work is needed to uncover the shape of equilibrium profiles. We show next that for any structure there exists a specialized equilibrium profile. Our proof is constructive and provides a method to find all specialized equilibria in general graphs. We use the following notions of graph theory. An *independent set* I of a graph $\mathbf{g}$ is a set of agents such that no two agents who belong to I are linked; i.e., $\forall i, j \in I$ such that $i \neq j$, $g_{ij} = 0$. An independent set is *maximal* when it is not a proper subset of any other independent set. Any maximal independent set I has the property that every agent either belongs to it or is connected to an agent who belongs to it.[6] For any agent i, there exists a maximal independent set I of the graph $\mathbf{g}$ such that i belongs to I.[7] This notably implies that any graph possesses at least one maximal independent set.

Maximal independent sets are a natural notion in our context. Because efforts are strategic substitutes, in equilibrium no two specialists can be linked. Hence, specialized equilibria are characterized by this structural property of a graph:

Proposition 3. *A specialized profile is a Nash equilibrium if and only if its set of specialists is a maximal independent set of the social structure* $\mathbf{g}$*. Since for every* $\mathbf{g}$ *there exists a maximal independent set, there always exists a specialized Nash equilibrium.*

The next example illustrates the concept of maximal independent sets and the relationship to specialized equilibria using our three basic graphs.

Example 2. *Specialized Equilibria and Maximal Independent Sets. In a complete graph, an independent set can include at most one agent. Hence, for* $n = 4$ *there are four specialized equilibria, corresponding to each agent. On the star, there are two maximal independent sets: the agent at the center, and the three agents in the periphery. These two sets correspond to the two specialized equilibria (and only equilibria) for the star (as seen in Figure 2 (b)). In the circle, there are two maximal independent sets, each containing two agents on opposite sides of the circle. Again, these two sets correspond to the specialized equilibria for the circle (as seen in Figure 2 (c)).*

C. Welfare Analysis

Here we explore the societal costs and benefits of different Nash equilibria. To gain a basic understanding, we take a standard utilitarian approach. We specify social welfare of profile $\mathbf{e}$ for

[6] To see this, suppose not. Let I be a maximal independent set, and let i be an agent who does not belong to I and is not connected to any agent who belongs to I. Then the set $I \cup \{i\}$ is an independent set, and hence I is not maximal.

[7] To see this, note that i itself is an independent set. To build a maximal independent set, begin with i and successively add agents not linked to i, then agents not linked to those agents, etc.

a graph $\mathbf{g}$ as the sum of the payoffs of the agents:

$$W(\mathbf{e};\mathbf{g}) = \sum_{i\in N} b\left(e_i + \overline{e}_i\right) - c\sum_{i\in N} e_i$$

where recall $\overline{e}_i$ is the sum of the search efforts of i's neighbors.

We first confirm the effect of externalities in our model. We say a profile $\mathbf{e}$ is *efficient* for a given structure $\mathbf{g}$ if and only if there is no other profile $\mathbf{e}'$ such that $W(\mathbf{e}';\mathbf{g}) > W(\mathbf{e};\mathbf{g})$. Since agents do not gain from the benefits of experimentation that accrue to their neighbors, for any non-empty graph, no Nash equilibrium profile is efficient.

Proposition 4. *When at least one pair of agents is linked, no Nash equilibrium profile is efficient.*

We therefore take a second-best approach. We ask which equilibrium profiles yield highest welfare, given a social structure $\mathbf{g}$. An equilibrium profile $\widetilde{\mathbf{e}}$ is *second-best* for a given structure $\mathbf{g}$ if and only if there is no other equilibrium profile $\widetilde{\mathbf{e}}'$ such that $W(\widetilde{\mathbf{e}}';\mathbf{g}) > W(\widetilde{\mathbf{e}};\mathbf{g})$. We denote the welfare associated with a second-best profile as $\widetilde{W}(\mathbf{g})$.

We write welfare to highlight the costs and benefits of different equilibria. Following Proposition 1, in any equilibrium, each agent has benefits of at least e^* information. Hence, $nb(e^*)$ is the minimum aggregate benefits in any equilibrium. In equilibria where some agents do not experiment, they have the benefits of more than e^* information. The benefits of such information are equal to $\sum_{j:e_j=0}[b(\bar{e}_j) - b(e^*)]$ where the summation is over all agents j who do not experiment. We can therefore express the welfare of an equilibrium $\mathbf{e}$ as the sum of three terms:

$$W(\mathbf{e};\mathbf{g}) = nb(e^*) + \sum_{j:e_j=0}[b(\bar{e}_j) - b(e^*)] - c\sum_i e_i. \tag{2}$$

where the second term is the *information premium* that can arise from specialization.

In (2), we see a trade-off between information premia and experimentation costs. Distributed equilibria yield no information premia. Specialization yields information premia but at possibly higher cost. The resolution of this trade-off depends on the shape of the graph. When the graph is such that specialists can be sufficiently well-connected, specialization can yield higher aggregate welfare. The next example illustrates. We then provide a general result.

Example 3. *Welfare Analysis. Consider the circle for $n = 4$. There are two kinds of equi-*

libria - distributed and specialized. In the distributed equilibrium each agent exerts $\frac{1}{3}e^*$. There is no information premium and total welfare is $4b(e^*) - \frac{4}{3}ce^*$. In any specialized equilibrium, there are two specialists located at opposite sides of the circle, and total welfare is $2b(e^*) + [2b(2e^*) - 2b(e^*)] - 2ce^*$. The specialized equilibrium yields an information premium of $2b(2e^*) - 2b(e^*)$ but entails higher costs. The specialized equilibrium will yield higher overall welfare when $[b(2e^*) - b(e^*)] > \frac{1}{3}ce^*$.

In general, the welfare comparison across equilibria depends on the value of information above e^*, embodied in the information premium. Let

$$\sigma \equiv \frac{b(ne^*) - b(e^*)}{c(n-1)e^*}$$

measure the value of this "extra information." Since the benefit function is increasing and concave, σ is always between 0 and 1, and higher σ means information above e^* is more valuble. Our result shows that when σ is sufficiently high, and agents who experiment are sufficiently well-connected, the information premium can exceed the costs.

Proposition 5. *Consider a graph* $\mathbf{g}$ *and two Nash equilibria* $\mathbf{e}^1$ *and* $\mathbf{e}^2$ *associated with* $\mathbf{g}$. *There exists a value of extra information* $\bar{\sigma} < 1$ *such that for any benefit functions with* $\sigma > \bar{\sigma}$, $W(\mathbf{e}^1; \mathbf{g}) > W(\mathbf{e}^2; \mathbf{g})$ *when* $\sum_i k_i e_i^1 > \sum_i k_i e_i^2$.

For instance, when $\mathbf{e}$ is a specialized equilibrium with a corresponding set of specialists I, we have $\sum_i k_i e_i = \left(\sum_{i \in I} k_i\right) e^*$. To compare the welfare of this equilibrium to the welfare of another specialized equilibrium, we can simply count the number of links between specialists and non-specialists.

D. The Benefits and Losses from New Links

In this section we examine the welfare effects of adding a new link to a graph. A new link has two, countervailing, effects. A link allows for greater access to new information. But an agent with greater access to information has less incentive to experiment. We show that this disincentive can lead to a loss in welfare.

Consider a graph $\mathbf{g}$ and two agents i and j who are not linked in $\mathbf{g}$. Denote by $\mathbf{g}+ij$ the graph obtained by connecting i and j in $\mathbf{g}$. We say that the link leads to a “loss in welfare” when the second-best level of welfare for graph $\mathbf{g} + ij$ is lower than that for $\mathbf{g}$; that is, $\widetilde{W}(\mathbf{g}) > \widetilde{W}(\mathbf{g} + ij)$. Let $\mathbf{e}$ be a second-best equilibrium profile for the social structure $\mathbf{g}$. There are two cases. First, in $\mathbf{e}$, either i or j does not experiment. In this case, $\mathbf{e}$ is also an equilibrium for $\mathbf{g} + ij$ and hence $W(\mathbf{e}; \mathbf{g} + ij) \geq W(\mathbf{e}; \mathbf{g})$. Second, in $\mathbf{e}$, both i and j experiment. In this case, $\mathbf{e}$ is not an equilibrium for the structure $\mathbf{g} + ij$. Adding a link between the two destroys the equilibrium pattern. When experimentation by both agents is required to secure high aggregate benefits, the new link can lead to a loss in welfare. We summarize these considerations as follows:

Proposition 6. *Consider a graph* $\mathbf{g}$ *and two agents* i *and* j *not linked in* $\mathbf{g}$. *A necessary condition to obtain a loss in welfare from linking* i *and* j *is that both agents experiment in all second-best equilibrium profiles on* $\mathbf{g}$.

We illustrate this proposition and the positive and negative effects of a new link in the following example.

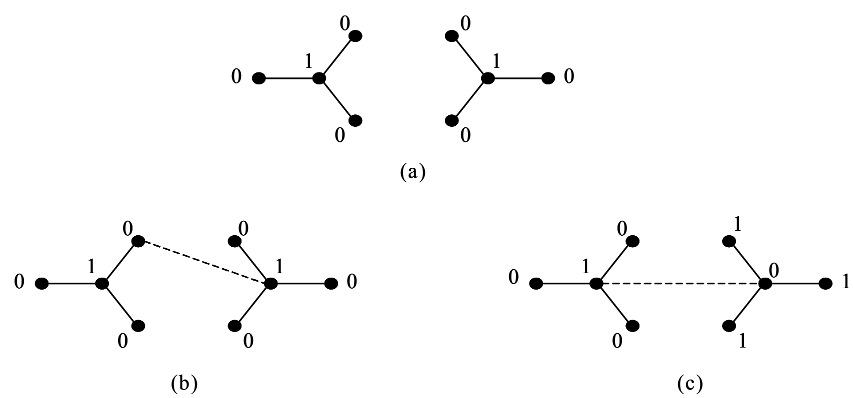

Figure 3. Benefits and Losses from a New Link - Equilibrium Outcomes

Example 4. *The Benefits and Losses from a New Link. Proposition 6 tells us to ask whether the two agents* i *and* j *experiment in all second-best patterns. Consider the two stars in Figure 3 panel (a). The Figure shows the unique second-best equilibrium pattern - both centers are specialists (see Appendix for proof). Connecting a peripheral agent to the center of the other star, as shown in Figure 3 panel (b), does not disrupt the equilibrium. The link thus increases welfare. In contrast, connecting the two centers, as shown in Figure 3 panel (c), destroys the equilibrium pattern and thus can decrease welfare. With the link between the two centers, the* *second-best equilibrium profiles involve the center of one star and peripheral agents of the other star as specialists. Welfare falls if the increased costs exceed the added information premium:* $2ce^* > b(4e^*) - b(e^*)$.

E. Equilibrium Selection: The Stability of Nash Equilibria

For any graph, there are potentially many Nash equilibria - ranging from specialized to distributed.[8] To gain some insight into the robustness of different outcomes, we consider a simple notion of stability based on Nash tâtonnement, see e.g. Fudenberg and Tirole (1996, p.23-25). Formally, an equilibrium $\mathbf{e}$ is stable if there exists a positive number $\rho > 0$ such that for any vector $\boldsymbol{\varepsilon}$ satisfying $\forall i, |\varepsilon_i| \leq \rho$ and $e_i + \varepsilon_i \geq 0$ the sequence $\mathbf{e}^{(n)}$ defined by $\mathbf{e}^{(0)} = \mathbf{e} + \boldsymbol{\varepsilon}$ and $\mathbf{e}^{(n+1)} = \mathbf{f}(\mathbf{e}^{(n)})$ converges to $\mathbf{e}$.

This standard notion has a strong selecting power in our setting. Only specialized equilibria are stable. Our result relies on the strategic substitability of efforts of linked agents. Consider an equilibrium where everyone does some experimentation, and decrease the effort of an individual i by a small amount. His neighbor(s) will adjust by increasing their own efforts. This increase can lead i to reduce his effort even more. The initial equilibrium is not stable. This process does not work in specialized equilibria when every agent j who does no experimentation is linked to two specialists. If we reduce the experimentation of these specialists, agent j will not adjust. He has access to two sources of information, and a small reduction will not lead him to increase his own effort.

The following result characterizes stable equilibria on any graph.

Proposition 7. *For any social structure* $\mathbf{g}$*, an equilibrium is stable if and only if it is specialized and every non specialist is connected to (at least) two specialists.*

The following example illustrates.

Example 5. *Stable Equilibria. Consider the star graph and the equilibrium where the center experiments* e^* *and peripheral agents do no experimentation. Let* $\rho > 0$ *be any small number and suppose that the center experiments* $e^* - \rho$*. In a first step, peripheral agents reply by each exerting* ρ*. This leads the center, in a second step, to decrease his experimentation to* $e^* - 3\rho$*. The initial*

[8] We show in section V below that multiplicity of equilibria is robust to alternative specifications of the model.

perturbation is amplified, and the equilibrium is not stable. In contrast, consider the equilibrium where all peripheral agents experiment. The only non-specialist (the center) is connected to three specialists, hence this equilibrium is stable. For instance, if peripheral agents each experiment $e^* - \rho$ and ρ is small enough, the center's best-reply is still to do no experimentation. At the second step, peripheral agents are back at their initial e^* play.

All of our results above can modified to describe stable equilibria rather than Nash equilibria. We can conduct welfare comparisons and analyze the effect of cutting links. We can show, for example, that even when restricting attention to stable equilibria, adding a link can reduce welfare.

IV. Specific Models of Social Networks

In this section we build stylized models of different social structures. We build three families of graphs, where each family represents a different organization of individuals in social and geographic space. Each family captures key interactions described in sociological and other literature. We ask how social structures affect experimentation.

A. Overlapping Neighborhoods

We first model an equitable social structure where each individual knows only a subset of the population, as in the circle above. We allow a person's set of neighbors to grow, and thereby capture increasing possibilities of communication across social and geographic space. Consider the following model: agents are arranged along a circle and numbered from 1 to n.[9] Each agent has k neighbors on the left and k neighbors on the right. We call such a structure an Overlapping Neighborhoods Graph.

We will see in our analysis our main themes. First, the network structure shapes equilibrium patterns. Second, specialized equilibria have benefits, and benefits of specialization increase as society becomes more integrated. But, third, society should not be completely integrated. Adding links is not always welfare-enhancing.

[9] The circle ($k = 1$) provides a standard way to represent local interactions, see e.g. Ellison (1993) and Eshel et. al. (1998). The family of graphs we consider in this section include the circle and also allows for expanding neighborhoods.

We first solve for the set of Nash equilibria for these graphs. We characterize specialized and distributed equilibria; hybrid equilibria are (loosely speaking) a mixture of the two.

Overlapping Neighborhoods Equilibria In an Overlapping Neighborhood Graph of size k,

(1) A strategy profile $\mathbf{e}$ is a distributed equilibrium if and only if there exists a common divisor of n and $2k+1$ - which we denote m - such that the sequence of the first m effort levels $(e_1, ..., e_m)$ satisfies $\sum_{i=1}^{m} e_i = \frac{m}{2k+1} e^*$ and the profile $\mathbf{e}$ is a repetition of this sequence every m agents. In particular, the profile where every agent experiments $\frac{1}{2k+1} e^*$ is an equilibrium ($m = 1$). (2) A strategy profile is a specialized equilibrium if and only if the distance between two consecutive specialists is at least $k + 1$ and no more than $2k + 1$.

The following example illustrates for graphs with $n = 12$ agents, shown in Figure 4. We show how the set of equilibria changes when we move from $k = 1$ to $k = 2$.

Example 6. *Equilibria in Overlapping Neighborhood Model. In panel (a), effort is distributed among all the agents, and each agent in the same position within each group of $2k + 1$ agents undertakes the same fraction of effort. In panel (b), specialists conduct all the experimentation. Notice that the specialists are at least $k + 1$ distance apart. In panel (c), we see a hybrid equilibrium. Again, the specialists must be at least $k + 1$ distance away from any other agent that does positive effort.*

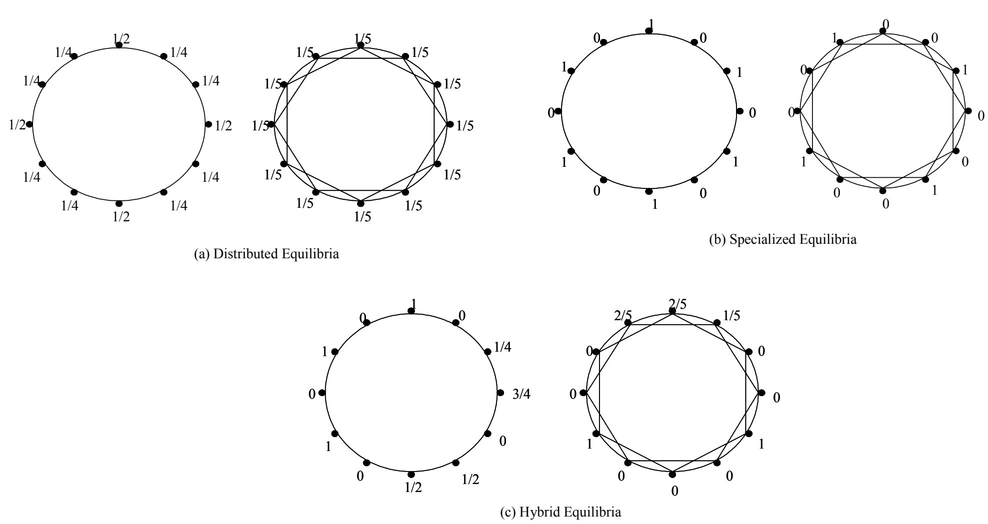

Figure 4: Equilibria in Overlapping Neighborhood Model

We explore the costs and benefits of specialized equilibria. We first show that specialization can yield higher welfare. Consider a graph where $k < \frac{n}{2} - 1$, so that the graph is not complete.[10] In all distributed equilibria, welfare is $nb(e^*) - \frac{n}{2k+1}e^*$. In specialized equilibria, the highest benefits are achieved when there are many specialists. The largest possible number of specialists is approximately $\frac{n}{k+1}$, in which case all non-specialists are connected to two specialists. Welfare is equal to $nb(e^*) + \left(n - \frac{n}{k+1}\right)[b(2e^*) - b(e^*)] - \frac{n}{k+1}ce^*$. The information premium of specialized equilibria exceeds the additional cost when[11]

$$b(2e^*) - b(e^*) > \frac{1}{2k+1}e^*c \tag{5}$$

With this inequality, we see that integration increases the benefits of specialization:

Integration increases Benefits of Specialization The relative benefits of specialized equilibria are increasing in k. If specialized equilibria yield higher welfare for some k', they also yield higher welfare for $k'' > k'$ (given $k'' < \frac{n}{2} - 1$).

The intuition is straightforward. As k increases, costs fall in both distributed and specialized equilibria. But in specialized equilibria, the information premium increases. We see this phenomenon in Figure 4 panel (b) above.

Despite these benefits of integration, welfare can fall when a graph becomes complete. When the graph is just less than complete, e.g., when $k = n/2 - 2$, specialized equilibria involve exactly two specialists and most people benefit from two sources of information. When the graph is complete, $k = n/2 - 1$, there can be at most one specialist; there is no possibility of an information premium. Hence, welfare can be higher when the graph is not complete.

Welfare Can Fall when the Graph is Complete Consider $k \leq n/2 - 2$. If the value of extra information is sufficiently high, then for $k' = n/2 - 1$, welfare of the second-best profile falls.

[10] In a complete graph, there is no comparison to make, since all equilibria yield the same level of welfare.

[11] We show in Appendix that the welfare of specialized equilibrium with $|I|$ specialists is equal to

$$W(|I|) = nb(e^*) + [(2k+1)|I| - n][b(2e^*) - b(e^*)] - c|I|e^*$$

Welfare is linear in the number of specialists and welfare of distributed equilibria equals $W(\frac{n}{2k+1})$. Therefore, if $b(2e^*) - b(e^*) > \frac{1}{2k+1}e^*c$, welfare is greatest when $|I|$ is greatest, while if $b(2e^*) - b(e^*) < \frac{1}{2k+1}e^*c$, welfare is greatest when $|I| = \frac{n}{2k+1}$ or for distributed equilibria.

B. Bridges between Communities

Our next model captures a social structure of communities, which are frequently observed patterns in societies.[12] People often learn from those in the same ethnic group, in the same village or in the same unit within a firm. We ask how links between such communities affects experimentation. Sociologists [e.g. Granovetter (1973), Burt (1992)] have long argued that links, or bridges, between communities increase opportunities for learning. In the analysis below we will see such positive effects. We will uncover, however, a new negative externality. When an agent gains access to a source of information outside her community, she may reduce her own experimentation.

For simplicity we consider only two communities.[13] Divide the population n into two sets, C_1 and C_2, where agents in each set are all linked to each other: $\forall i, j \in C_t, t = 1, 2, g_{ij} = 1$. Some agents in different sets are also linked: For some i and j, where $i \in C_1$ and $j \in C_2$, $g_{ij} = 1$. We call such agents *bridge agents* and call the link between them a *bridge*. To maintain the idea of distinct communities, we assume that at least one agent in each community has no links to agents in the other community. Let B denote the set of bridges for a graph $\mathbf{g}$, and let β be the number of bridges. We call this structure a Community/Bridge Graph, and a particular graph is characterized by the number of its bridges β.

In our analysis we will again see the themes of the paper. First, the social structure shapes the set of equilibrium patterns. Bridge agents, who have more connections than other agents, experiment less on average than other agents. Second, there are benefits of specialization. When bridge agents are specialists, their efforts benefit many people. Finally, additional links between a bridge agent and the other community can reduce the bridge agent's incentive to experiment, and overall welfare can fall.

We first characterize the Nash equilibrium patterns for this social structure. We are concerned, in particular, how the number of bridges β affects the set of equilibria. We find that the greater the number of bridges, the greater the equilibrium efforts of non-bridge agents. Because bridge agents have access to information in the other community, in equilibrium at least one of each pair of bridge agents does no experimentation. Hence, the greater the number of bridges, across

[12] Mapping the cohesive subgroups present in real networks is a main issue of social network analysis. Sociologists use a variety of concepts and techniques to address this task, see e.g., chapter 7 in Wasserman and Faust (1994) and Girvan and Newman (2002) for recent methodological advances.

[13] Bala and Goyal (2001) look at the same type of structure in their study of conformism and diversity.

equilibria the average effort of non-bridge agents rises. We summarize these findings below, then illustrate the equilibria in Example 7.

Equilibria for Community-Bridge Graphs For a Community-Bridge graph with β bridges, the Nash equilibria are the profiles such that: (1) total effort exerted within each community is equal to e^*, and (2) for any two agents i and j connected by a bridge, at least one of agents i and j does no experimentation. As a consequence, across equilibria the average effort cost of non-bridge agents is higher for social structures with greater β.

Example 7. *Equilibrium Patterns for Community-Bridge Graphs. Consider two communities of four agents each, pictured in Figure 5. In panel (a), there are no bridges ($\beta = 0$), and any distribution of effort e^* in each community is a Nash equilibrium. E.g., equal distribution, $\frac{1}{4}e^*$ for each agent, is a Nash equilibrium. In panel (b), there is one bridge ($\beta = 1$). There is no equilibrium in which both bridge agents do strictly positive effort. Hence, as shown in the Figure, e^* is distributed among the non-bridge agents in one community, and total effort of non-bridge agents increases. In panel (c), where $\beta = 2$, there is another pair of agents for which it is not possible for them both to do positive effort. Hence, effort by non-bridge agents increases again.*

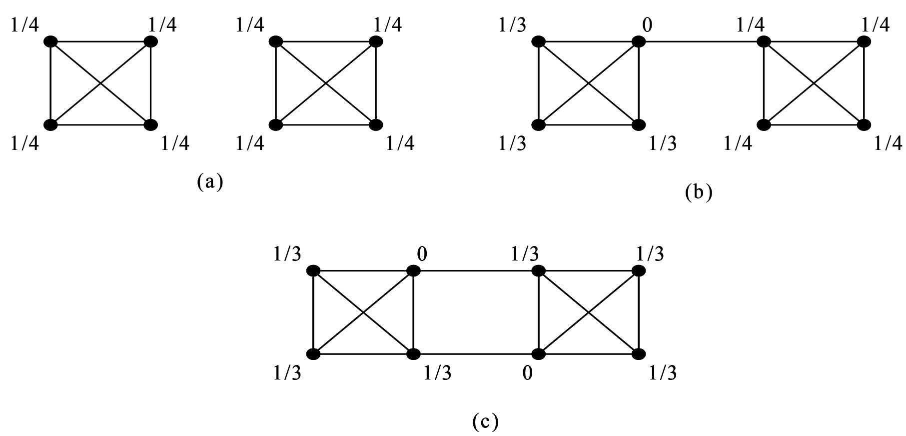

Figure 5. Equilibria in Communities Connected by Bridges

This result counters sociological wisdom concerning bridges, which are a central concept of the literature on information transmission. Links between communities, it is argued, allows information to spread across social boundaries [Granovetter (1973), Burt (1992), Watts and Strogatz (1998)]. Our result highlights a different aspect of bridges. When experimentation is costly and effort is endogenous, bridge agents experiment less, which increases the burden on other agents. Thus, our analysis points towards a more ambiguous vision of bridges.

[To reconcile our result with the sociological wisdom, we also consider the set of equilibria when information can pass from a member of a community, through a bridge, to a member of another community. When information diffuses two steps in the graph, we still see that bridge agents can reduce their effort. There are two types of Nash equilibria [see Appendix]: (1) bridge agents exert a total effort of e^* and other agents do no experimentation; (2) bridge agent do no experimentation and in each community, non-bridge agents exert a total effort of e^*. With two step diffusion, bridge agents either provide information to agents in both communities, or do no effort themselves and information just passes through them to the other community (as emphasized in the sociology literature). With two-step transmission, bridge agents can take even more advantage of their position to reduce their effort to zero. We now return to single-step diffusion.]

We consider the welfare of different equilibrium profiles. Here we see a tension: while bridge agents have less incentive to experiment than other agents, their effort is important for social welfare. If i is a bridge agent, we denote by N_i^B the set of neighbors of i in the other community. The welfare of an equilibrium profile $\mathbf{e}$ for a social structure $\mathbf{g}$ with a set of B bridges is then

$$W(\mathbf{e},\mathbf{g}){=}nb(e^*) + \left[\sum_{i\in B:e_i=0} b(e^* + \sum_{j\in N_i^B} e_j) - b(e^*)\right] - 2ce^* \tag{6}$$

where the second term is the information premium earned by those agents linked to bridge agents in the other community who experiment. It follows directly from this expression that the second-best equilibrium profile exclusively involves experimentation by bridge agents in one or, often, both communities:

Search Should be Concentrated among Bridge Agents Consider a graph with $\beta \geq 1$ bridges. In second best profiles, bridge agents in one community exert all the effort. In the other community, aggregate effort exerted by bridge agents is either 0 or e^*.

We can see this result in the graphs in Figure 6. In all of the pictured equilibria, bridge agents experiment. These equilibria yield higher welfare than equilibria in which no bridge agents experiment. In such equilibria, it is as if the communities are not linked by bridges, and each agent earns only $b(e^*)$ information.

Finally, we see how, despite the additional possibilities for learning, adding a new bridge can reduce overall welfare. We apply Proposition 6. We call a bridge between two agents a *separate bridge* if and only if neither of the two agents is linked to another agent in the other community. We find that adding separate bridges between communities always increases welfare. But the same is not true for *non-separate* bridges, which can reduce welfare. An agent who had an incentive to experiment when connected to a single agent in the other community may not experiment when linked to two. We illustrate the positive and negative effects of new bridges in Example 8.

Example 8. *The Effect of New Bridges on Welfare. Consider two communities of four agents each. In each of four panels in Figure 6, we illustrate a second best profile for* $\beta = 0, 1, 2, 3$ *separate bridges. The total costs are the same in each case, and the information premium increases from* 0 *to* $b(2e^*) - b(e^*)$ *to* $2\left[b(2e^*) - b(e^*)\right]$ *to* $\left[b(2e^*) + 2b(\frac{3}{2}e^*) - 3b(e^*)\right]$.

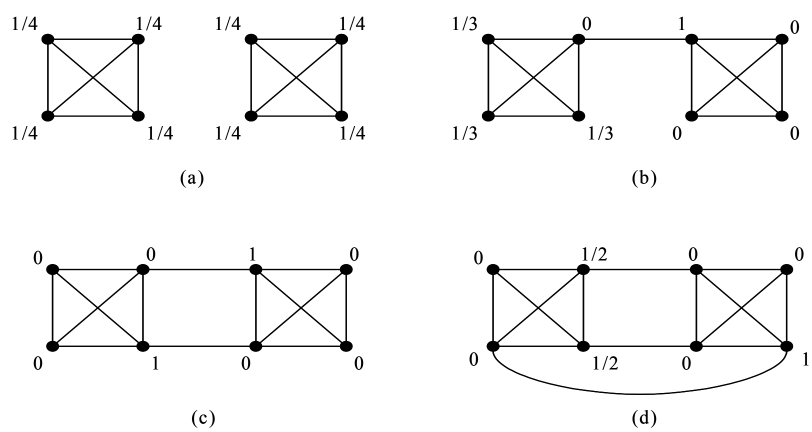

Figure 6. Second-Best Equilibria with Separate Bridges

We next show how a new non-separate bridge can decrease welfare. Consider the two communities with four agents each in Figure 7. Panel (a) shows a social structure with three bridge agents in each community. In any second-best equilibrium for this social structure, the agents with two bridge neighbors are specialists. Hence, applying Proposition 6, a link between them can reduce welfare. Indeed, we see in panel (b) when this link is added, in second-best equilibria only one of them is a specialist. Welfare falls. [Without the link, the information premium in a second-best equilibrium is $4[b(2e^*) - b(e^*)]$*, and with the link it is* $3[b(2e^*) - b(e^*)]$*].*

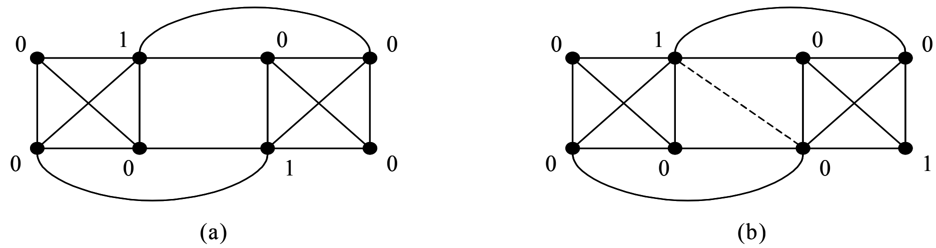

Figure 7. Negative Effect of New Non-Separate Bridge

C. Core-Periphery Graphs

We next construct and analyze graphs that represent core-periphery social structures.[14] Such structures consist of a core - a group of densely connected agents - and a periphery - a group of agents with connections only to the core. Research suggests core-periphery patterns characterize many social and economic interactions. In economic geography, manufacturing activity is often concentrated in one or few regions - the core - while agricultural activity is dispersed in the periphery and supplies the core [see e.g. Krugman (1991)]. Examples of core-periphery networks include the interlocking directorates of firms in the United States [Mintz and Shwartz (1981)], international trade [Snyder and Kick (1979)], and the social organization of academic research [Mullins et al. (1977)].

We model core-periphery structures as follows. We divide the population into two sets, the core, C, and the periphery, P, such that all agents in the core are linked to each other and no agents in the periphery are linked to each other; i.e., $\forall i,j \in C, g_{ij} = 1$ and $\forall i,j \in P$ *s.t.* $i \neq j, g_{ij} = 0$. Core agents may or may not have links to peripheral agents. We call core agents that have one or more links to peripheral agents *p-core agents*. To make the analysis interesting, we assume there is at least one p-core agent, so that at least one core agent has a link to the periphery. To simplify the exposition, throughout the analysis below we assume that no p-core agents have the same set of neighbors.[15]

Each core-periphery graph is distinguished by two measures which we derive to capture the

[14] Bramoullé (2002a, 2002b) develops and analyzes graphs with a core-periphery structure in a study of anti-coordination games.

[15] We have solved for all the Nash equilibrium profiles of core-periphery graphs, including those that do not satisfy this assumption. Moreover, the welfare results below hold whether or not this assumption is satisfied.

extent to which specific core agents are critical to agents in the periphery. For a p-core agent i, consider his neighbors in the periphery, $N_i \cap P$. Let r_i denote the number of links between these neighbors and core agents other than himself:

$$r_i = \sum_{j \in N_i \cap P} (k_j - 1)$$

We consider the minimum and maximum values for a graph $\mathbf{g}$: For the set of p-core agents, let $\underline{r} = \min_i\{r_i\}$ and $\overline{r} = \max_i\{r_i\}$. When $\underline{r}$ is small, for some agents in the periphery a single core agent is their sole source of information. When $\overline{r}$ is also small, this is true for most peripheral agents.

Our analysis explores how the structure shapes equilibrium patterns, the importance of specialization, and the effect of new links.

In any core-periphery graph, we find there are only two shapes of Nash equilibria, reflecting the division between the core and periphery. Both involve specialists. In one type, one p-core agent is a specialist. His information spreads to his neighbors in the periphery and to other core agents. In the second type, all peripheral agents are specialists. The latter outcome is consistent with the argument that major innovations often take place in the periphery of social systems.[16] We show the general result then illustrate in an Example.

Equilibria in Core-Periphery Graphs On any core-periphery graph, there are two types of Nash equilibria: (1) No p-core agents experiments, each peripheral agent exerts e^*, and the other core agents collectively exert a total effort of e^*. (2) One p-core agent is a specialist and experiments e^*, all his neighbors (the rest of core and his links in the periphery) do not experiment, and the remaining peripheral agents each exert e^*.[17]

Example 9. *Equilibrium Patterns in Core-Periphery Graphs. Consider the graphs in Figure 8. In the graphs on the left, three of the four core agents are p-core agents.*[18] *In the graphs on the*

[16] See Kuhn (1963) and Chubin (1976) on scientific innovations, and Mansfiel (1968), Jewkes et al. (1959), and Hamberg (1963) on industrial innovations.

[17] When all core agents have links to the periphery and one of them has exactly one peripheral neighbor, a slight variation on the second equilibrium exists. If i is a core agent and j his unique peripheral neighbor, the profile where $e_i + e_j = e^*$, all the other core agents do no search, and their peripheral neighbors search e^* is an equilibrium.

[18] *For ease of exposition the graphs in the Figure violate our assumption that no two core agents have the same set of neighbors. All of welfare results are the same whether or not this assumption is satisfied.*

right, all core agents have links to the periphery. Panel (a) shows equilibria where a p-core agent is a specialist. Panel (b) depicts the equilibria where all peripheral agents experiment. Notice the difference in panel (b) between the equilibria in the graph on the left and right: On the left, the core agent with no links to the periphery experiments, while on the right, all core agents have links to the periphery and do no experimentation.

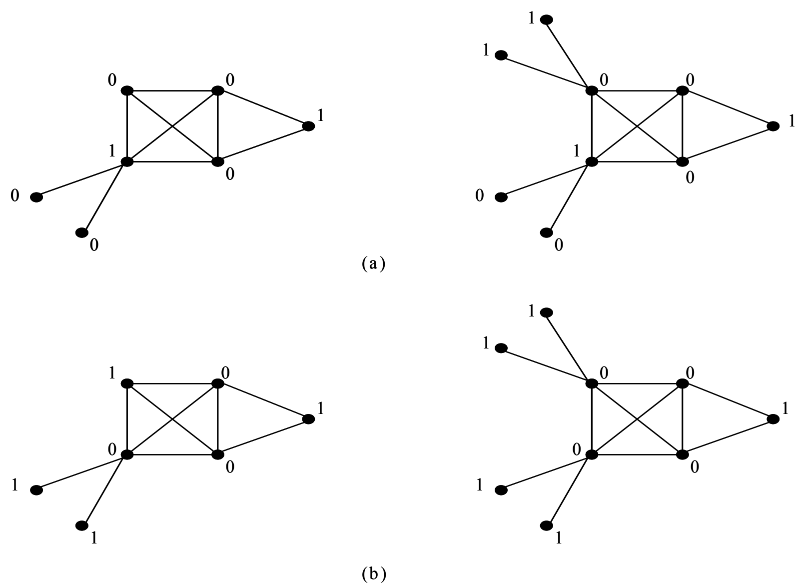

Figure 8. Equilibria in Core-Periphery Graphs

While there are two types of equilibria, our welfare analysis indicates that social welfare can be higher when p-core agents specialize. The pattern of links between the core and periphery, as captured in our measures $\underline{r}$ and $\overline{r}$, determines which of the two types of equilibria is second-best. We start with the simplest case of sparse links between the core and periphery. Suppose that any peripheral agent who has a link to the core has no other link; i.e., $\overline{r} = 0$.[19] In this case it is second best for an agent in the core to specialize.

Sparse Links between Core and Periphery Consider any core-periphery graph such that $\overline{r} = 0$. In every second-best profile, a p-core agent is a specialist and this agent is among those with the most links to the periphery.

[19] Recall we have assumed that at least one agent in the periphery has a link to an agent in the core.

This case, $\overline{r} = 0$, is quite similar to the star graph. Information generated in the periphery does not benefit many agents in the core. The information premium is always smaller than the additional costs, and therefore it is always better in terms of welfare for peripheral agents to experiment less.

Now suppose there are denser links between the core and periphery. Let at least one peripheral agent have more than one neighbor in the core; that is, $\overline{r} \geq 1$. In this case, it is possible that equilibria where all peripheral agents experiment is second-best. As we saw in section III above, second-best profiles generally trade-off effort costs and information premia. Here, p-core agents might receive an information premium, since they have access to information in the core as well as in the periphery. When $\underline{r}$ is high, many p-core agents' peripheral neighbors are well-connected to the core, and research by these peripheral agents brings more informational benefits. Hence, social welfare can be higher when experimentation is concentrated in the periphery. To prove this result, we apply the techniques of Proposition 5. For each type of equilibrium, we identify the maximal independent set of agents that constitutes the set of specialists. We then count the links between agents in the set and agents outside the set. We then compare these numbers across equilibria, and the comparison ultimately depends on the value of $\underline{r}$.

Dense Links between Core and Periphery Consider any core-periphery graph where $\overline{r} \geq 1$: (1) If all agents in the core are linked to agents in the periphery and if $\underline{r} < |C| - 1$, then there exists a value of extra information $\overline{\sigma} < 1$ such that for all $\sigma \geq \overline{\sigma}$, the second best equilibrium involves specialization by a p-core agent. If $\underline{r} > |C| - 1$ then there exists a value of extra information $\sigma' < 1$ such that for all $\sigma \geq \sigma'$ in the second best equilibrium all peripheral agents experiment. (2) If there is at least one core agent with no links to the periphery and if $\underline{r} = 0$, then there exists a value of extra information $\sigma'' < 1$ such that for all $\sigma \geq \sigma''$ the second best equilibrium involves specialization by a p-core agent. If $\underline{r} \geq 1$, there is a value of extra information $\sigma''' < 1$ such that for all $\sigma \geq \sigma'''$ such that peripheral experimentation is second best.

We illustrate in the following example.

Example 10. *Core versus Periphery Experimentation. Consider the graphs in Figure 8 and first compute the coefficients r_i. We obtain $r_i = 1$ for the core agents with a unique peripheral neighbor* *and $r_i = 0$ for the core agents with two peripheral neighbors. Thus, $\underline{r} = 0$ and our result above applies. In both graphs, if the value of extra information is sufficiently high, in the second-best profile a core agent with two neighbors in the periphery is a specialist. Calculations confirm this result. For instance, consider the graph on the left in panel (a). The welfare of this equilibrium is $5b(e^*) + 2b(2e^*) - 2ce^*$. The equilibrium where a different p-core agent is a specialist yields a welfare of $6b(e^*)+b(3e^*)-3ce^*$, which is always lower. The equilibrium where all peripheral agents are specialists, shown on the left in panel (b), yields a welfare of $4b(e^*) + 2b(2e^*) + b(3e^*) - 4ce^*$. The difference in welfare between this equilibrium and the best equilibrium with p-core agent experimentation is $b(3e^*) - b(e^*) - 2ce^*$, which is always negative.*

Finally, we consider the impact of adding links to a core-periphery structure. We apply Proposition 6: a link between an agent i and an agent j will always increase welfare unless agents i and j experiment in every second-best profile for the original graph. One important case is when a core agent is a sole source of information to a set of peripheral agents. A link between this core agent and an agent in the periphery who provides information to the core can reduce welfare. The example below illustrates.

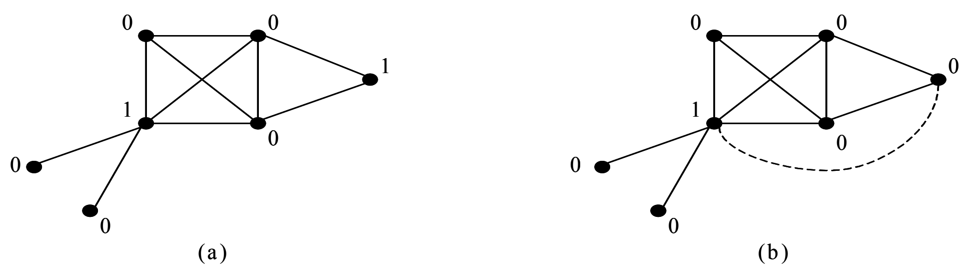

Figure 9. Effect of New Link Between the Core and Periphery

Example 11. *Effect of a New Link between the Core and the Periphery. Consider the graph in panel (a) Figure 9. We showed in the previous example that the equilibrium where a p-core agent with two peripheral neighbors is a specialist is always second-best and yields a welfare of $W = 5b(e^*) + 2b(2e^*) - 2ce^*$. As described above, linking this agent with the third peripheral agent might lower social welfare. To see this, examine the possible equilibria on the new graph in panel (b): (1) In the equilibrium shown in the Figure, the core agent with three neighbors in the* *periphery is a specialist and $W_1 = 7b(e^*) - ce^*$; (2) Another equilibrium would involve a different p-core as specialist, yielding welfare $W_2 = 6b(e^*) + b(3e^*) - 3ce^*$, which is always lower than W_1; (3) The last possible equilibrium involves all peripheral agents experimenting, yielding welfare $W_3 = 4b(e^*) + 2b(2e^*) + b(4e^*) - 4ce^*$. Therefore, the second-best level of welfare falls with the new link if $W > W_1$ and $W > W_3$. That is, if $\frac{1}{2}[b(4e^*) - b(e^*)] < ce^* < 2[b(2e^*) - b(e^*)]$.*

V. Robustness of Results

In this section, we discuss the robustness of our findings to changes in the model's specifications. We have explored several different directions including convex costs, heterogeneity, and information diffusion.

Convex costs.

First, suppose that costs of experimentation $c(e_i)$ are convex instead of linear. Clearly, convex costs drive the outcome towards effort sharing. For example, on the complete graph, there is now a unique equilibrium where all individuals experiment the same amount. Despite this effect, we see that multiple specialized equilibria still emerge on certain incomplete graphs like the circle. More generally, Proposition 3 extends as follows. There usually exists a positive integer s such that a specialized profile is a Nash equilibrium if and only if its set of specialists is a maximal independent set *and* each non specialist is connected to at least s specialists. This result implies that multiple specialized equilibria emerge on *infinitely* many graphs. In terms of welfare, specialization still generates information premia. Despite the convexity of the cost function, these benefits can be sufficiently high so that specialization yields higher welfare. This strongly confirms our findings that networks may foster specialization and that specialization may have welfare benefits.

Heterogeneity.

Next, suppose that agents are heterogeneous in their benefits from information $b_i(e_i + \bar{e}_i)$ and costs of experimentation $c_i e_i$.[20] We can use many of our techniques to analyze this case. Equilibrium outcomes can be represented by an idiosyncratic threshold effort level e_i^* such that in equilibrium, $e_i = 0$ when $\bar{e}_i \geq e_i^*$ and $e_i = e_i^* - \bar{e}_i$ otherwise. Individuals with higher benefits

[20] Heterogeneity arises from a simple model of impure altruism. Suppose individual i obtains a 'warm glow' [Andreoni (1990)] of me_i for each neighbor that benefits from her experimentation e_i. This gain effectively lowers i's marginal cost of $c - k_i m$. Individuals with more links earn greater pleasure from generating new information and, hence, have lower effective costs of experimentation.

or lower costs have higher thresholds. On the complete graph, there is a (generically) unique equilibrium where the individual with highest threshold does all the experimentation. Heterogeneity leads to specialization. This is clearly reinforced on incomplete networks. We can adapt the proof of Proposition 3 to show that on any graph there exists a specialized equilibrium where the individual with highest threshold experiments. Furthermore, *multiple* specialized equilibria emerge on certain incomplete graphs. Which equilibrium yields higher welfare is, again, determined by a trade-off between information premium and effort costs. This shows the robustness of our insights.

Longer information diffusion and decay.

Finally, individuals could learn experimental results from further away in a network. For instance, suppose that experimentation results diffuse k steps without decay. Our setting can easily be adapted to this case. Define the graph $\mathbf{g}^{(k)}$ as follows: $g_{ij}^{(k)} = 1$ if i and j are less than k-step apart in $\mathbf{g}$, and 0 otherwise. Under the assumption that agents can discern redundant information, k-step diffusion on the graph $\mathbf{g}$ is formally equivalent to 1-step diffusion on $\mathbf{g}^{(k)}$ and our analysis directly extends. We can also study comparative statics with respect to k. For instance since $\mathbf{g}^{(k)}$ has more links than $\mathbf{g}$, our results on the addition of new links apply and the highest level of welfare attainable on $\mathbf{g}^{(k)}$ might in fact be lower than on $\mathbf{g}$. Indeed, this outcome is precisely our result in the overlappings neighborhoods model.

Information decay would complicate the analysis. If experimental results by one agent eventually reaches *all* other agents in society, we suspect that specialization might not emerge in equilibrium. Heterogeneity in efforts caused by strategic substitutability and network asymmetries, however, will certainly persist. The assumption that information travels infinitely far in a network, however, seems quite unrealistic. Friedkin's (1983) study of communication networks finds just the opposite; information diffuses only one or two steps. In the more sensible case where information diffuses a few steps with decay specialization is still guaranteed on certain incomplete graphs.

VI. Conclusion

This paper introduces a network model of strategic experimentation. We explore how the social or geographic links affects the pattern of experimentation and have three main findings.

First, social networks induce specialization and inequalities. We find that on any network there exist specialized equilibria where some agents experiment and others free-ride. Certain structures, like the star, admit only specialized equilibria. An agent's position in the graph affects his equilibrium action: agents who have a more peripheral position, or who are isolated in their own community, exert relatively more effort, on average, in equilibrium. The study of alternative specifications of the model, and notably of convex costs, confirms the robustness of these effects. Specialization and inequality in experimentation emerge from the interplay between strategic substitutability and networks. And due to strategic substitutability, specialized equilibria alone can be stable.

Second, we show that inequality in experimentation can be socially desirable. Strategic substitutability limits the total level of experimentation attainable in equilibrium. Specialization can then be welfare-enhancing. Two conditions must be met: the value of extra information must be sufficiently high and specialists are linked to sufficiently many people. These conditions point to a conflict between individual behavior and aggregate social welfare when agents have different numbers of neighbors - as in the communities-bridges and core-periphery graphs. Agents who have more links exert comparatively less effort in many equilibria. But second-best profiles involve experimentation by these well-connected agents, whose experimentation would lead to the highest social benefits.

Third, we show that new links can lower social welfare. In general, there are two effects of new links. A link allows more transmission of information, but greater access to information reduces individual incentives to experiment. We find this negative effect can dominate and have aggregate and distributional consequences. For instance, contrary to the prevailing wisdom, we show that bridges between communities can be detrimental to others. Agents who gain outside sources of information can reduce their own efforts. As for aggregate effects, when two agents are in critical positions in a network, a link between them can lower their experimentation incentives and reduce overall welfare.

Descriptive and sociological studies suggest that social structures affect experimentation and information diffusion.[21] Our analysis thus points to potentially important determinants of exper-

[21] See, for example, Gladwell's (2000) descriptive account. People who have extensive knowledge of the marketplace are usually quite peripheral. Rogers (1995) reports similar findings when contrasting the social position of "innovators," who typically are the first to experiment, to that of "early adopters," who rely on innovators'

imentation and innovation. For instance, we expect individuals who have active social neighbors to have high benefits but experiment less. We also expect individuals who have prominent social positions to bear less of the experimentation costs, and instead to rely on others' efforts. This analysis could potentially guide empirical research. For instance in the study of new crop adoption in developing countries, one could combine the investigation of strategic effects as in Foster and Rosenzweig (1995) with network data of the type collected in Conley and Udry (2002).

Future theoretical research could investigate network formation and how information transmission feeds back into the evolution of social links. In our analysis, we look at existing networks; channels of information transmission are already in place when the need for new information appears. Thus, our setting is probably more appropriate to understand short-run patterns of experimentation and information diffusion within established networks. Looking at long-term issues and network evolution provides a promising avenue for future research.

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experiences before making their decisions: "early adopters are a more integrated part of the local social system than are innovators," (p. 263).

APPENDIX
Corollaries mentioned in Example 1.

Corollary 1. *When the graph* $\mathbf{g}$ *is complete, a profile* $\mathbf{e}$ *is a Nash equilibrium if and only if* $\sum_i e_i = e^*$.

Proof: Let i be an agent who exerts positive effort $e_i > 0$. Then, $e_i + \bar{e}_i = e^*$. Since i is connected to all other individuals, it means that $\bar{e}_i = \sum_{j \neq i} e_j$, and $\sum_j e_j = e^*$. Reciprocally, these profiles are obviously Nash equilibria. ■

Corollary 2. *On the star graph, a profile* $\mathbf{e}$ *is a Nash equilibrium if and only if: either (1) the agent in the center plays* e^* *and all the other agents play* 0*; or (2) the agent in the center plays* 0 *and all the other agents play* e^*.

Proof: The two profiles are clearly Nash equilibria. We show that they are the only ones. Suppose the center agent i exerts a effort level $0 < e' < e^*$. In equilibrium, each of his neighbor's must experiment $e^* - e'$. Suppose now each of agent i's neighbors experiment $e^* - e'$. Agent i's information will then equal $(n-1)(e^* - e') + e'$ which is strictly greater than e^* for $n > 2$. Hence, agent i would have an incentive to deviate and lower her effort. ■

The Corollary for the circle is found in Section IV below.

Proof of Proposition 7.

Our proof relies on the following lemma.

Lemma 1. *If* $\mathbf{e} \leq \mathbf{e}'$, *then* $\mathbf{f} \circ \mathbf{f}(\mathbf{e}) \leq \mathbf{f} \circ \mathbf{f}(\mathbf{e}')$.

Proof: Suppose that $\forall i, e_i \leq e'_i$. Then, $e^* - \bar{e}_i \geq e^* - \bar{e}'_i$, hence $\max(e^* - \bar{e}_i, 0) \geq \max(e^* - \bar{e}'_i, 0)$ and $\mathbf{f}(\mathbf{e}) \geq \mathbf{f}(\mathbf{e}')$. Applying $\mathbf{f}$ again to this inequality yields the result. ■

Consider first an equilibrium that is not purely specialized and denote by $I = \{i : 0 < e_i < e^*\}$. Let $\rho > 0$ be a small number and define a perturbation $\boldsymbol{\varepsilon}$ as follows: $\forall i \in I, \varepsilon_i = \rho$ and $\forall i \notin I, \varepsilon_i = 0$. Introduce the matrix h such that $h_{ij} = g_{ij}$ if $i \neq j$ and $h_{ii} = 0$. If ρ is small enough, we have $\bar{\varepsilon}_i \leq e_i$, hence $\mathbf{f}(\mathbf{e} + \boldsymbol{\varepsilon}) = \mathbf{e} - h\boldsymbol{\varepsilon}$ where the product $h\boldsymbol{\varepsilon}$ denotes the usual product of the matrix h by the vector $\boldsymbol{\varepsilon}$. Then, $\mathbf{f} \circ \mathbf{f}(\mathbf{e} + \boldsymbol{\varepsilon}) = \mathbf{e} + h^2\boldsymbol{\varepsilon}$. Since $(h^2)_{ii} = k_i \geq 1$, we obtain $(h^2\varepsilon)_i \geq \rho = \varepsilon_i$ and $h^2\boldsymbol{\varepsilon} \geq \boldsymbol{\varepsilon}$. Therefore, $\mathbf{f} \circ \mathbf{f}(\mathbf{e} + \boldsymbol{\varepsilon}) \geq \mathbf{e} + \boldsymbol{\varepsilon}$. By applying the lemma we can see that for any finite number k, $\mathbf{f}^{(2k)}(\mathbf{e} + \boldsymbol{\varepsilon}) \geq \mathbf{e} + \boldsymbol{\varepsilon}$, which is strictly greater than $\mathbf{e}$. Therefore, the sequence of best-responses never converges back to $\mathbf{e}$ and the equilibrium is not stable.

Consider next a specialized equilibrium $\mathbf{e}$ such that i is a non specialist who is connected to a unique specialist j. Let $\rho > 0$ be a small number and define a perturbation $\boldsymbol{\varepsilon}$ as follows: $\varepsilon_i = \rho$ and $\varepsilon_l = 0$ if $l \neq i$. Then, clearly, $e_l^{(1)} = e_l$ for any l except j and $e_j^{(1)} = e^* - \rho$. Next, $e_l^{(2)} = e_l$ for any l except for neighbors of j whose only specialist neighbor is j. These agents, which include i, all play ρ. This means that $\mathbf{f} \circ \mathbf{f}(\mathbf{e} + \boldsymbol{\varepsilon}) \geq \mathbf{e} + \boldsymbol{\varepsilon}$ and we can apply the same argument as above, hence this equilibrium is not stable.

Finally, let us prove that specialized equilibria in which every non-specialist is connected to (at least) two specialists are stable. Take $\mathbf{e}$ such an equilibrium, let I be the set of specialists in $\mathbf{e}$, and let $\delta = \frac{1}{n^2}e^*$. Consider any perturbation $\boldsymbol{\varepsilon}$ such that $\forall i, |\varepsilon_i| < \delta$ and $\varepsilon_i + e_i \geq 0$. First, determine $\mathbf{e}^{(1)}$. For $i \notin I$, $\bar{e}_i^{(0)} = \bar{e}_i + \bar{\varepsilon}_i = |N_i \cap I|e^* + \bar{\varepsilon}_i$. Since $|\bar{\varepsilon}_i| \leq n\delta \leq e^*$, this implies that $\bar{e}_i^{(0)} \geq e^*$, hence $e_i^{(1)} = 0$. Despite the perturbation, non-specialists still do no effort. For $i \in I$, $\bar{e}_i^{(0)} = \bar{\varepsilon}_i$, hence $e_i^{(1)} = e^* - \bar{\varepsilon}_i$. Next, determine $\mathbf{e}^{(2)}$. For $i \notin I$, $\bar{e}_i^{(1)} = |N_i \cap I|e^* - \sum_{j \in N_i \cap I} \bar{\varepsilon}_j$.

Since $|\sum_{j\in N_i\cap I}\bar{\varepsilon}_j|\leq n^2\delta\leq e^*$, we obtain again that $e_i^{(2)}=0$. Finally, for $i\notin I$, $\bar{e}_i^{(1)}=0$ hence $e_i^{(2)}=e^*$. We showed that $\mathbf{f}\circ\mathbf{f}(\mathbf{e}+\boldsymbol{\varepsilon})=\mathbf{e}$, hence $\mathbf{f}^{(k)}(\mathbf{e}+\boldsymbol{\varepsilon})=\mathbf{e}$ for all $k\geq 2$, hence the equilibrium is stable.

Proof of Proposition 4.

Let $\mathbf{e}$ be an equilibrium and i an agent such that $e_i>0$ and for some $j\neq i, g_{ij}=1$. From Proposition 1, we know that $\overline{e}_i+e_i=e^*$, which implies $b'(\overline{e}_i+e_i)=c$. Examine the partial derivative of the welfare function with respect to e_i.

$$\frac{\partial W}{\partial e_i}=\sum_{j\in i\cup N_i}b'(\sum_{k\in j\cup N_j}e_k)-c=\sum_{j\in N_i}b'(\sum_{k\in j\cup N_j}e_k)$$

When b is strictly concave, this derivation implies $\frac{\partial W}{\partial e_i}>0$, and welfare could be strictly improved by increasing e_i. Agents do not internalize the benefits their efforts have for others.

Proof of Proposition 5

Consider an equilibrium $\mathbf{e}$ and i such that $e_i=0$. Since b is increasing and concave, we have $\sigma c(\bar{e}_i-e^*)\leq b(\bar{e}_i)-b(e^*)\leq c(\bar{e}_i-e^*)$. This means that

$$\sigma c[\sum_{i:e_i=0}(\bar{e}_i-e^*)-\sum_i e_i]+(1-\sigma)c\sum_i e_i\leq W(\mathbf{e},\mathbf{g})-nb(e^*)\leq c[\sum_{i:e_i=0}(\bar{e}_i-e^*)-\sum_i e_i]$$

In addition, $\sum_{i:e_i=0}(\bar{e}_i-e^*)=\sum_{i\in N}(e_i+\bar{e}_i)-ne^*$. By switching the double summation, we then obtain $\sum_{i\in N}(e_i+\bar{e}_i)=\sum_i\sum_j g_{ij}e_j=\sum_j\sum_i g_{ij}e_j=\sum_j(k_j+1)e_j$. Substituting yields

$$\sigma c\sum_j k_je_j+(1-\sigma)c\sum_i e_i\leq W(\mathbf{e},\mathbf{g})-n[b(e^*)-ce^*]\leq c\sum_j k_je_j$$

Hence, as σ tends to 1, $W(\mathbf{e},G)-n[b(e^*)-ce^*]$ tends to $c\sum_j k_je_j$. (We consider benefit functions for which $b'(e^*)=c$ so that equilibria are not affected by changes in σ). Therefore, there exists a threshold $\bar{\sigma}<1$ such that if $\sigma>\bar{\sigma}$, $\sum_j k_je_j^1>\sum_j k_je_j^2$ implies that $W(\mathbf{e}^1,\mathbf{g})>W(\mathbf{e}^2,\mathbf{g})$.

Calculations for example 4.

For a star with n agents, the welfare of the equilibrium where the center experiments is $W^1=nb(e^*)-ce^*$ while the welfare of the equilibrium where peripheral agents effort is $W^2=(n-1)[b(e^*)-ce^*]+b((n-1)e^*)$. Their difference is equal to $W^1-W^2=(n-2)ce^*-[b((n-1)e^*)-b(e^*)]$ which is positive.

Equilibria on overlapping neighborhood graphs.

Consider first a distributed equilibrium such that for every i, $0<e_i<e^*$. It follows from Proposition 1 that for every i, $\sum_{j=i-k}^{i+k}e_j=e^*$. Subtracting the equation for i and the equation for $i+1$ yields $e_{i-k}-e_{i+k+1}=0$. Thus for every i, $e_i=e_{i+2k+1}$. Recall that when the indice becomes greater than n, one simply substracts n from it. This shows that, more generally, $e_i=e_j$ as soon as there exists two integers t,t' such that $i-j=(2k+1)t-nt'$. Define m as the greatest common divisor of $2k+1$ and n. A standard result of arithmetics is that there always exist two integers t and t' such that $m=(2k+1)t-nt'$. This means that for every i, $e_i=e_{i+m}$ and the whole effort profile is generated by the first m individual efforts $e_1,...,e_m$.

Consider, next, a specialized equilibrium. Let I be the set of agents that exert e^*. This set must be non-empty (otherwise, some agent would increase his effort). By Proposition 1, in a Nash equilibrium, no agent $j\in I$ can be linked to any other agent in I. Hence, the distance between each agent $j\in I$ must be at least $k+1$. The distance can also not be more than $2k+1$. If it were, then the agent $k+1$ away would be learning no information from her neighbors and hence would have an incentive to deviate and exert positive effort. It is evident that these strategies constitute a Nash equilibrium.

Welfare analysis on overlapping neighborhoods graphs.

Let $\mathbf{e}$ be an equilibrium. We know that $\sum_{i\in N}(e_i+\sum_{j\in N_i} e_j)=\sum_{j\in N}(k_j+1)e_j$ and also, by Proposition 1, that $\forall i, e_i+\sum_{j\in N_i} e_j \geq e^*$. Since on overlapping neighborhoods, $\forall j, k_j=2k$, we have $\sum_{j\in N} e_j \geq \frac{n}{2k+1}e^*$ which shows that distributed equilibria yield lowest effort costs.

Consider next a specialized equilibrium with $|I|$ specialists. No agent can be connected to three specialists, since in this graph, it would mean that two specialists are linked. Denote by n_1 the number of agents connected to a single specialist and by n_2 the number of agents connected to two specialists. With these notations, welfare can be expressed as $W=nb(e^*)+n_2[b(2e^*)-b(e^*)]-c|I|e^*$. We then compute n_2. Clearly, $|I|+n_1+n_2=n$. Next, we count the number of links between specialists and non-specialists. On the one hand, it is equal to $\sum_{i\in I} k_i=2k|I|$ since each specialist has k_i links with non-specialists. On the other hand, it is equal to n_1+2n_2. Thus, $n_1+2n_2=2k|I|$. Hence, $n_2=(2k+1)|I|-n$. Welfare only depends on the number of specialists in the specialized equilibrium.

Equilibria on bridge/communities graphs.

Our proof relies on the following lemma.

Lemma 2. *Let $\mathbf{e}$ be an equilibrium for a graph $\mathbf{g}$. Consider two agents i and j such that $i\cup N_i\subset j\cup N_j$. If $e_j>0$, then for every agent $k\neq i$ who belongs to $N_j\backslash N_i$, $e_k=0$.*

Proof**:** When $e_j>0$, by Proposition 1, $e_j+\sum_{k\in N_j} e_k=e^*$. The information obtained by j is equal to the sum of the information obtained by i and the information produced by the neighbors of j who do not communicate with i

$$e_j+\sum_{k\in N_j} e_k = e_i+\sum_{k\in N_i} e_k + \sum_{k\neq i\in N_j\backslash N_i} e_k$$

The Nash condition on i implies that $e_i+\sum_{k\in N_i} e_k\geq e^*$ which means that $\forall k\neq i\in N_j\backslash N_i, e_k=0$ and $e_i+\sum_{k\in N_i} e_k=e^*$. ■

Now, consider a bridge connecting i and j and denote by i_0 an agent in i's community who is not a bridge agent. Since $i_0\cup N_{i_0}\subset i\cup N_i$, we can apply Lemma 2. Either $e_i=0$ or all the agents to whom i is connected who are not neighbors of i_0 do no effort. This means that if $e_i>0$, all the bridge agents in the other community who are connected to i do no effort. Reciprocally, the profiles described in the Proposition are evidently Nash equilibria.

Equilibria for two-step diffusion.

Bridge agents are connected to every agent in the graph $\mathbf{g}^2$. Suppose that one bridge agent j exerts strictly positive effort $e_j>0$ and consider i_1 a non-bridge agent in one community and i_2 a non-bridge agent in the other community. Since $i_1\cup N_{i_1}\subset j\cup N_j$ and $i_2\cup N_{i_2}\subset j\cup N_j$, we can apply Lemma 2 to $i_1$and j and i_2 and j. For every s belonging to $N_j\backslash N_{i_1}$ or to $N_j\backslash N_{i2}$, $e_s=0$, which implies that all non-bridge agents do no effort. Therefore, bridge agents must exert a total effort of e^*, and any such profile is an equilibrium. In contrast, if bridge agent do no effort, non-bridge agents in each community do not have access to information outside their community, hence the Nash equilibria are the profiles such that non-bridge agents in each community exert a total effort of e^*.

Welfare analysis on bridge-community graphs.

Consider an equilibrium profile $\mathbf{e}$. If $\forall i \in B, e_i = 0$, the information premium equals zero, hence welfare is strictly greater when some bridge agent exerts positive effort. Hence, in any second-best profile $\tilde{\mathbf{e}}$ there exists an agent $i \in B$ such that $\tilde{e}_i > 0$. Now suppose that in $\tilde{\mathbf{e}}$ some effort is done by non-bridge agents in i's community. That is, $\exists j \notin B$ such that $g_{ij} = 1$ and $e_j > 0$. Consider the profile $\mathbf{e}'$ defined as follows: $e'_j = 0$, $e'_i = e_i + e_j$ and $e'_k = e_k$ for every $k \neq i, j$. The effort done by j in $\tilde{\mathbf{e}}$ is transferred upon i in $\mathbf{e}'$. Then, $\mathbf{e}'$ is an equilibrium profile in which bridge neighbors of i receive greater informational benefits than in $\tilde{\mathbf{e}}$. Thus $W(\mathbf{e}'; \mathbf{g}) > W(\tilde{\mathbf{e}}; \mathbf{g})$ which is contradictory. This shows that in second-best profiles, as soon as some bridge agents do some effort in one community, they do all the effort.

Effect of separate bridges

We describe the shape of second-best profiles when bridges are separate. Specifically, we show that an equilibrium yields highest welfare when: If β is even, $\beta/2$ agents in each community effort the same amount; If β is odd, $(\beta - 1)/2$ agents in one community effort equally, and the same holds for $(\beta + 1)/2$ agents in the other community. When $\beta = 1$, the result is a direct consequence of the previous result. Suppose that $\beta \geq 2$. Our proof proceeds in two steps: (1) We determine the equilibria that yield greatest welfare, conditional on the hypothesis that t bridge agents in one community and $\beta - t$ bridge agents in the other community exert positive effort; and (2) We use the previous calculations to determine the optimal t. First, suppose that t bridge agents in one community exert a total effort of e^*. These agents are denoted by $1, ..., t$. How should we allocate the effort to maximize the information premium received by their neighbors in the other community? This optimization problem can be expressed as follows:

$$\max_{e_1 + ... + e_t = e^*} \sum_{i=1}^{t} b(e_i + e^*) - b(e^*)$$

Since b is concave, the solution to this problem is standard and involves equal sharing of effort. That is, an optimal solution is such that $\forall i, e_i = \frac{1}{t}e^*$. Next, compute the optimal t. For clarity, denote by $\pi(t) = b((1 + \frac{1}{t})e^*) - b(e^*)$. The greatest welfare in equilibrium with t is $W(t, \beta) = t\pi(t) + (\beta - t)\pi(\beta - t)$ if $0 < t < \beta$ and $W(0, \beta) = \beta\pi(\beta)$. This last case can easily be eliminated. Since b is increasing, we have $\pi(\beta) < \pi(\beta - 1)$ and $\pi(\beta) < \pi(1)$, which means that $\beta\pi(\beta) < (\beta - 1)\pi(\beta - 1) + \pi(1) = W(1, \beta)$. Therefore, in any second-best profile, $0 < t < \beta$ and the optimization problem reduces to

$$\max_{t \in \{1,2,...,\beta-1\}} t\pi(t) + (\beta - t)\pi(\beta - t)$$

One can check that the function $t \mapsto t\pi(t)$ is increasing and concave,[22] which implies that the optimal t is an integer closest to $\beta/2$.

Effect of non-separate bridges

We characterize the second-best profiles for the graph of Example 8 (see Figure 7). First, consider the graph without the link. The profile depicted in figure 7 yields an information premium of $4[b(2e^*) - b(e^*)]$. We know that effort is exclusively done by bridge agents in either one community, or both communities. When bridge agents in a single community effort, the three bridge agents in the other community benefit from their information, hence the information premium cannot be greater than $3[b(2e^*) - b(e^*)]$ and it is not second-best. When a bridge agent

[22] The first derivative equals $b((1 + \frac{1}{t})e^*) - b(e^*) - \frac{1}{t}e^* b'((1 + \frac{1}{t})e^*)$ and is positive because b is concave, while the second derivative equals $\frac{1}{t^3}(e^*)^2 b''((1 + \frac{1}{t})e^*)$ and is negative.

with a single bridge experiments, his neighbor in the other community does not effort, hence an agent with single bridge in the other community experiments as well. Hence, the two bridge agents with two bridges do no effort, and the information premium equals $2[b(2e^*)-b(e^*)]$. Second, consider the graph with the link. Either a bridge agent i with three bridges effort, and it is best when $e_i = e^*$, which yields an information premium of $3[b(2e^*) - b(e^*)]$. Or only bridge agents with single bridges effort and the information premium is $2[b(2e^*) - b(e^*)]$.

Equilibria in core-periphery graphs.

Consider an equilibrium profile $\mathbf{e}$. Clearly, total effort done on the core $\sum_{i \in C} e_i$ cannot be greater than e^*. Suppose that there is a p-core agent i who exerts positive effort, but who is not a specialist: $0 < e_i < e^*$. Recall, $N_i \cap P$ denotes the set of peripheral neighbors of i. We distinguish two cases: $|N_i \cap P| = 1$ and $|N_i \cap P| \geq 2$.

(1) Suppose that i has a unique peripheral neighbor j. Since $j \cup N_j \subset i \cup N_i$, Lemma 2 tells us that agents in $C \backslash N_j$ do no effort. N_j is the set of core agents connected to j, to which i belongs. There are two cases. (a) $e_j = 0$. In this case, i receives all her information from agents in N_j, and hence the total effort done on N_j is exactly equal to e^*. This is possible only when agents of N_j who effort do not have other peripheral neighbors than i. (Since total effort done on N_j is e^*, these other peripheral neighbors could not effort, hence would receive less than e^*). All core efforters have the same, unique peripheral neighbor. (b) $e_j > 0$. In this case, $e_j + e_i + \sum_{N_j \backslash \{i\}} e_k = e^*$. If j has other core neighbors than i, j must be their unique peripheral neighbor. Again, all core experimenters have the same, unique peripheral neighbor.

(2) Suppose that $|N_i \cap P| \geq 2$. Take $j \in N_i \cap P$. Since $j \cup N_j \subset i \cup N_i$, Lemma 2 yields $\forall k \in N_i \backslash N_j, e_k = 0$. Thus, all other peripheral neighbors of i do no effort. The same argument applied to another peripheral neighbor of i implies that $\forall k \in N_i \cap P, e_k = 0$ and all peripheral neighbors of i do no effort. Thus, for any j peripheral neighbor of i, the total effort done by her neighbors other than i must be at least equal to $e^* - e_i$. Since these agents all belong to the core, it means that their total effort is exactly equal to $e^* - e_i$. Thus, any core experimenter must be connected to all the neighbors of i and cannot have peripheral neighbors not connected to i. That is, all core experimenters must have the same peripheral neighbors.

When core agents have distinct neighbors, the only case where a p-core agent exerts positive effort but is not a specialist is when this agent has a unique peripheral neighbor, who does not have other neighbors, and all core agents are connected to the periphery (as described in the Footnote). In general, we showed that there are only two additional possibilities with respect to the set of equilibria described in the text: (1) When several core agents have a unique and common peripheral neighbor and when all core agents are connected to the periphery, any profile such that e^* is allocated in any way among these core agents and their peripheral neighbor, other core agents do zero and other peripheral agents are specialists is an equilibrium; and (2) When several core agents have the same peripheral neighbors, any profile where these core agents exert a total effort of e^*, their common peripheral neighbors do no effort, other core agents do no effort, and other peripheral agents are specialists is an equilibrium.

Welfare analysis: sparse links.

Denote by $\mathring{C}$ the set of p-core agents. For each p-core agent j, denote by $k_{j,P}$ the number of peripheral neighbors of j. That is, $k_{j,P} = k_j - (|C| - 1)$. We examine two cases. First, suppose that some core agent is not connected to the periphery. Consider an equilibrium where all peripheral agents are specialists. Peripheral agents earn $b(e^*) - ce^*$, while the p-core core agent j earns $b[(k_{j,P} + 1)e^*]$. Welfare in equilibrium equals

$$W_0 = nb(e^*) - ce^* + \sum_{j\in\mathring{C}} \{b[(k_{j,P}+1)e^*] - b(e^*) - k_{j,P}ce^*\}$$

Consider next the equilibrium where i is a p-core specialist. Core agents who are not connected to the periphery earn $b(e^*)$. Other p-core agent j earns $b[(k_{j,P}+1)e^*]$, while the payoff of i is $b(e^*) - ce^*$. Peripheral agents not connected to i earn $b(e^*) - ce^*$, while peripheral neighbors of i earn $b(e^*)$. In the end, welfare equals

$$W_i = nb(e^*) - ce^* + \sum_{\substack{j\in\mathring{C}\\ j\neq i}} \{b[(k_{j,P}+1)e^*] - b(e^*) - k_{j,P}ce^*\}$$

Substracting both formulas yields $W_i - W_0 = -\{b[(k_{i,P}+1)e^*] - b(e^*) - k_{i,P}ce^*\}$ which is positive and increasing with $k_{i,P}$ since b is concave and $b'(e^*) = c$. Second, suppose that all core agents are connected to the periphery. The expression for W_i does not change, hence is increasing in $k_{i,P}$. Welfare of an equilibrium where core agents do no effort is $W_0' = nb(e^*) + \sum_{j\in\mathring{C}}\{b[k_{j,P}e^*] - b(e^*) - k_{j,P}ce^*\}$ since total effort is equal to zero on the core. Again, substracting both expressions yields

$$W_i - W_0' = -\{b(k_{i,P}e^*) - b(e^*) - (k_{i,P}-1)ce^*\} + \sum_{\substack{j\in\mathring{C}\\ j\neq i}} \{b[(k_{j,P}+1)e^*] - b(k_{j,P}e^*)\}$$

which is positive.

Welfare analysis: dense links.

First, note that any equilibrium has a welfare less than or equal to the welfare of a specialized equilibrium (even when several core agents have the same set of neighbors). Thus, to determine second-best profiles we only need to determine which specialized equilibrium yields greatest welfare. To do this, we compute for all specialized equilibria with maximal independent set of specialists I the number of links between specialists and non-specialists $\sum_{i\in I} k_i$. Consider first the equilibrium where p-core agent j is a specialist. Here, the set of specialists is composed of j and all peripheral agents not connected to j. Since $k_j = |C| - 1 + |N_j \cap P|$, we obtain $\sum_{i\in I} k_i = |C| - 1 + |N_j \cap P| + \sum_{i\in P, i\notin N_j} k_i$. Using the parameter c_j, this expression can be simplified to $\sum_{i\in I} k_i = |C| - 1 + \sum_{i\in P} k_i - c_j$. Next, consider the equilibrium where all peripheral agents are specialists. Here we have to distinguish two cases: (1) There is a core specialist (not connected to the periphery) and $\sum_{i\in I} k_i = |C| - 1 + \sum_{i\in P} k_i$; (2) All core agents are connected to the periphery and $\sum_{i\in I} k_i = \sum_{i\in P} k_i$. Combining these expressions and Proposition 5 leads to the results described in the text, except for the case when $\underline{c} = 0$ and all core agents are connected to the periphery. In this last case, direct examination of welfare yields the result. Some core agents are such that all their peripheral neighbors are only connected to them. effort by one of these core agents with most neighbors yields greatest welfare. (Computations are similar to the case of the star graph.)

Convex costs.

The formulation of Bergstrom et al. (1986) provides an example of convex experimentation costs. Individuals allocate their income y between experimentation e_i and private good consumption x_i. With separable preferences, $u_i(\mathbf{e}, x_i) = b(e_i + \bar{e}_i) + a(x_i)$. If the price of private good is normalized to 1 and the public good price is p, the income constraint becomes $x_i + pe_i = y$. The marginal cost of experimenting is now equal to $pa'(y - pe_i)$. It captures both the direct cost and the foregone consumption of private goods and is generally increasing in e_i.

With convex experimentation costs $c(e_i)$, individual i's best-response is to play 0 if $b'(\bar{e}_i) \leq c'(0)$ and to play e_i such that $b'(e_i + \bar{e}_i) = c'(e_i)$ otherwise. On the complete graph, there is a unique equilibrium where the effort of any individual is e such that $b'(ne) = c'(e)$. To extend Proposition 3, define the number s as the smallest integer such that $b'(se^*) \leq c'(0)$ where e^* is the effort level in isolation. It is well-defined as soon as $c'(0) > b'(\infty)$. For instance, $p = 1$ and $a \equiv b$ in the previous model yield $s = 2$. On the circle with 4 individuals, the two maximal independent sets give rise to two distinct specialized equilibria.

Finally, to illustrate the welfare analysis with convex costs, consider again the case where $p = 1$ and $a \equiv b$. Either $\bar{e}_i < y$ and $e_i = \frac{1}{2}(y - \bar{e}_i)$ and $u_i = 2b[\frac{1}{2}(y + \bar{e}_i)]$, or $\bar{e}_i \geq y$ and $e_i = 0$ and $u_i = b(\bar{e}_i) + b(y)$. Individuals who do no search earn more than individuals who search, hence specialization still yield information premium. To see that the benefits from specialization might be greater than the added costs, consider the following core-periphery graph: three agents in the core and four agents in the periphery who are connected to every core agent. The profile where each peripheral agent searches $\frac{1}{2}y$ and agents in the core do no search is a specialized equilibrium on this graph. Its yields a welfare of $W^s = 8b(\frac{1}{2}y) + 3[b(y) + b(2y)]$. Any distributed equilibrium would yield a welfare lower than or equal to $W^d = 14b(y)$, because $b[\frac{1}{2}(y + \bar{e}_i)] \leq b(y)$ when $e_i > 0$. Specialization yields greater welfare as soon as $W^s > W^d$ which is equivalent to $b(2y) - b(y) > \frac{4}{3}[b(y) - b(\frac{1}{2}y)]$, which is clearly possible.

Heterogeneity.

We next construct a specialized equilibrium on any network $\mathbf{G}$. Order agents through decreasing thresholds $e_1^* \geq ... \geq e_n^*$. Construct a maximal independent set I as follows. Include individual 1 in I. Then remove 1 and all her neighbors. Add to I the remaining agent with highest threshold. Then, remove her neighbors and repeat the operation til no agent is left. The profile where each agent i in I searches e_i^* while others do no search is a specialized equilibrium.

Next, consider 4 agents located on the circle and suppose that $e_1^* > e_2^* > e_3^* > e_4^*$. If the thresholds are not too different, there are two equilibria: $(e_1^*, 0, e_3^*, 0)$ and $(0, e_2^*, 0, e_4^*)$.

Longer diffusion.

We show that, even with decay, certain graphs still yield specialized equilibria. Suppose that information diffuses two steps with decay δ. The set of neighbors of i is still N_i, while $N_i^{(2)}$ now denotes the set of individuals two steps away from i. The level of information obtained through others is now $\bar{e}_i = \delta \sum_{j \in N_i} e_j + \delta^2 \sum_{j \in N_i^{(2)}} e_j$. The best-response of individual i is still $f_i(e) = \max(e^* - \bar{e}_i, 0)$. Next, consider the circle with 6 agents. One can see that the profile where two opposite individuals search e^* and others do no search is an equilibrium when δ is high enough (specifically, when $\delta + \delta^2 \geq 1$).

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